PHYS 434 Optics

Lecture 1: **Course Introduction**, Waves and Electromagnetism in a Nutshell

General Information

TERM: Winter, 2019 – January 7 to April 12

LECTURE TIME: MW, 2:35pm to 3:55pm

LOCATION: Rutherford RPHYS 115

CREDITS: 3

INSTRUCTOR INFORMATION:

Name: Dr. Vanessa Graber

Contact: vanessa.graber@mcgill.ca

Office: MSI 207 (3550 McGill University)

Office hours: TBD (doodle poll in week 1)

General Information

TEACHING ASSISTANTS:

Rigel Zifkin, rigel.zifkin@mail.mcgill.ca, RPHYS 420
Yang Lan, yang.lan2@mail.mcgill.ca, RPHYS 406
Ziggy Pleunis, ziggy.pleunis@mail.mcgill.ca, MSI 010
Office hours: TBD (doodle poll in week 1)
Weekly tutorial: TBD (doodle poll in week 1)

HANDS-ON DEMONSTRATIONS COORDINATOR:

Dr. Kelly Lepo, kelly.lepo@mcgill.ca, WONG 0160

General Information

COMMUNICATION:

- Best to contact me via email or during office hours.
- Course announcements will be sent through the MyCourses system (you are expected to know the content of email announcements within one weekday of receiving them).
- Lecture notes, homework problems, exam review, and supplementary materials will also be posted on the MyCourses website.
- Please make sure to carefully read through the syllabus and calendar after this lecture.

Course Overview

PREREQUISITES:

- PHYS 342 Majors or PHYS 352 Honours EM Waves
- Be comfortable with mathematical techniques used to describe waves, including the wave equation, complex numbers, multivariate calculus, and Fourier methods.

COURSE CONTENT:

- PHYS 434 will introduce fundamental concepts of optics, the mechanisms behind optical devices and applications, and give some insight into modern developments.

Course Overview

COURSE CONTENT:

- PART I – Review of Electromagnetism and Light Propagation: index of refraction, scattering, light propagation in media, reflection, refraction (2 weeks)
- PART II – Geometric Optics: mirrors and lenses, optical systems, aberrations (2 – 3 weeks)
- PART III – Superposition, Polarisation and Interfe-rence: coherence, polarisation, scattering, optical activity, interference/interferometers (3 – 4 weeks)

Course Overview

COURSE CONTENT:

- PART IV – Diffraction, Fourier Optics and Modern Optics: diffraction and its applications, Fourier methods, Gaussian beams, holography (5 weeks)
- Where appropriate, we will discuss numerical approaches using e.g. Python, Mathematica, Matlab.
- More details about the course content together with important dates can be found in the course calendar.

Course Overview

EVALUATION:

- Your final grade will be determined by homework assignments (regular problem sets plus hands-on demonstrations) (50%), a midterm exam (20%) and the final exam (30%).

PROBLEM SETS:

- Problem sets (6 in total) will be posted online approximately every two weeks. The due date, which will be specified for each problem set, will typically be at the beginning of class one week after posting.

Course Overview

PROBLEM SETS:

- Problem sets will include exercises (questions, which you should work through but do not need to be hand in) and problems (graded and must be handed in).
- One question on each of the two exams will be very closely related to an assigned problem or exercise.

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PROBLEM SETS:

- Problem sets will include exercises (questions, which you should work through but do not need to be hand in) and problems (graded and must be handed in).
- One question on each of the two exams will be very closely related to an assigned problem or exercise.
- You are encouraged to collaborate with other students on the problem sets. However, the solutions that you hand in at the end must reflect your own work.
- The use of solution sets in graded homework is plagiarism and will be treated accordingly.

Course Overview

PROBLEM SETS:

- Late problem sets will not be accepted unless an extension has been approved by me or one of the TAs prior to the due date. To compensate for this, the lowest problem set grade will be dropped, if it helps your final grade.

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HANDS-ON DEMONSTRATIONS:

- You will carry out 3 hands-on demonstrations in groups of three, scheduled approximately once per month. For due dates see the course calendar.

Course Overview

HANDS-ON DEMONSTRATIONS:

- Each demo will include preparing the material, performing experiments and writing a report.
- Group compositions will be randomly assigned and changed for each demo. We will form the first set of groups at the beginning of Lecture 2, Jan 9.

Course Overview

HANDS-ON DEMONSTRATIONS:

- Each demo will include preparing the material, performing experiments and writing a report.
- Group compositions will be randomly assigned and changed for each demo. We will form the first set of groups at the beginning of Lecture 2, Jan 9.
- The demos will be run by Dr. Kelly Lepo and each group will have to schedule a 90min slot (outside of class time) for each experiment with her.
- Slots are on a first come, first served basis, so you are encouraged to book a time as early as possible.

Course Overview

HANDS-ON DEMONSTRATIONS:

- A TA will be there to assist you during the experiment.
- 1 week after your scheduled demo time, each group has to hand in a report, which will be graded by a TA.

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MIDTERM EXAM:

- There will be one closed-book 80 minute midterm exam during regularly scheduled class time on Wednesday, Feb 20.
- You will be permitted to use an 8x11" equation sheet (one side), a dictionary, and a calculator.

Course Overview

MIDTERM EXAM:

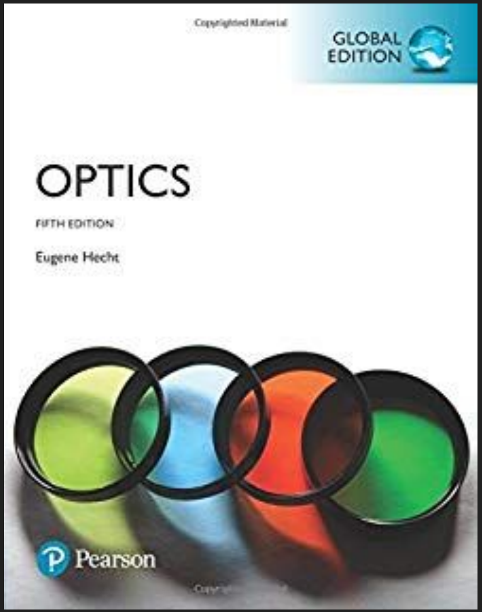
- If you miss the exam due to a documented medical emergency, your midterm grade will be replaced by the average of your homework and final exam grades.
- Following the midterm, you have the opportunity to write a research paper to replace half of your midterm grade (composed of a proposal and a short article discussing a research topic in modern optics).

Course Overview

FINAL EXAM:

- There will be a closed-book 3hour final exam (date to be determined). You will be permitted to use an 8x11" equation sheet (two sides), dictionary, and calculator.
- If you are unable to write your final exam due to a serious, documented reason (e.g., illness), you may apply for a deferral. If your application is accepted, you will be permitted to write the final exam during the next deferred exam period.

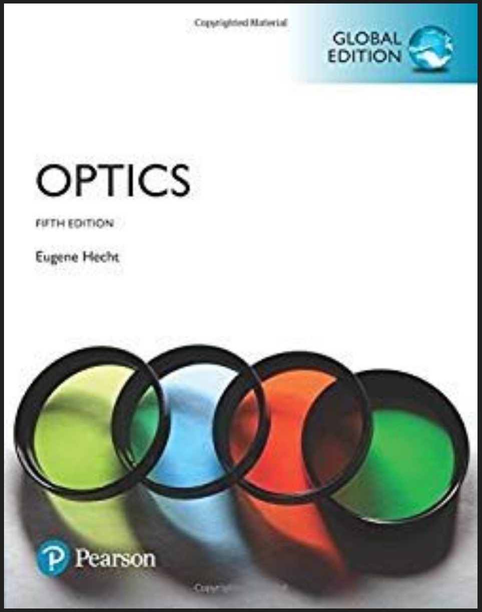
Course Overview



READING MATERIALS:

- The required text for the course is ´Optics` by Eugene Hecht.
- I am using the 5th edition, but earlier editions also cover the course material in an acceptable manner.
- The 5th edition is available at bookstores or can be purchased online but is not cheap (note that the digital version is however much cheaper than the printed one), so you may wish to look for used copies or take advantage of copies in the Schulich library.

Course Overview



READING MATERIALS:

- This text can be quite wordy, but contains a lot of detail, background material and exercises you may find helpful.
- There are many other books, which could be useful, e.g. ´The Light Fantastic - A Modern Introduction to Classical and Quantum Optics` by Ian Kenyon.
- Please let me know ASAP, if you have trouble accessing the main book for this course!!!!!!

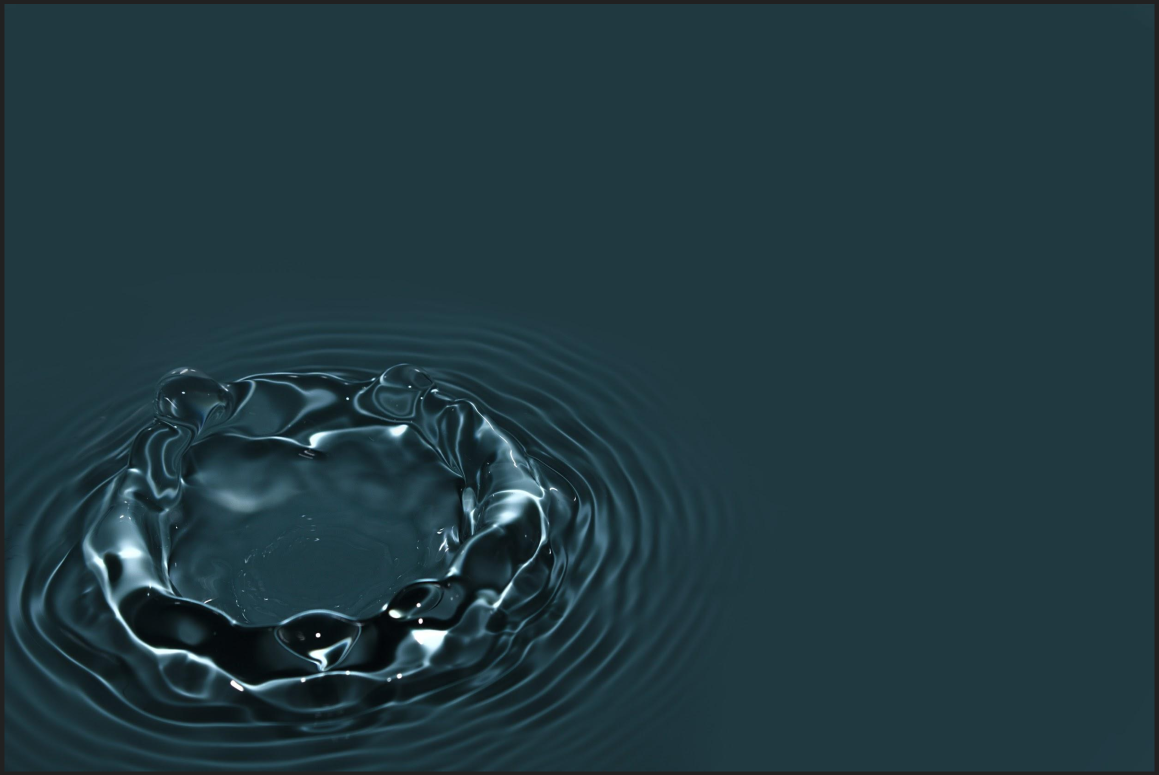
Course Overview

Please consult the syllabus and course calendar also during the term, if you have any questions concerning the course administration.

QUESTIONS???

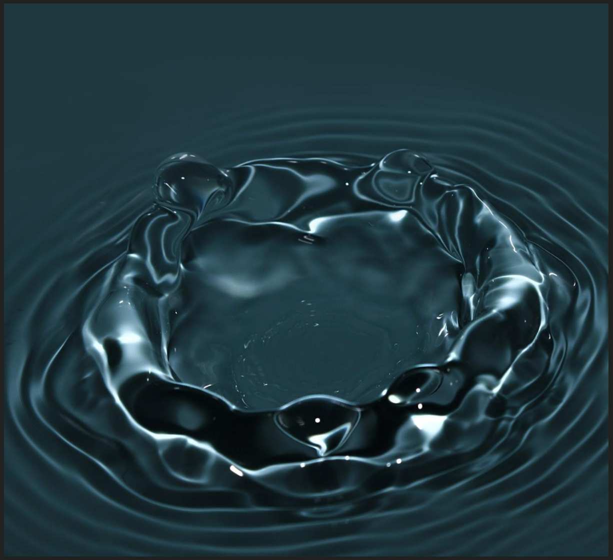
PHYS 434 Optics

Lecture 1: Course Introduction,
Waves and Electromagnetism in a Nutshell



Nature of light

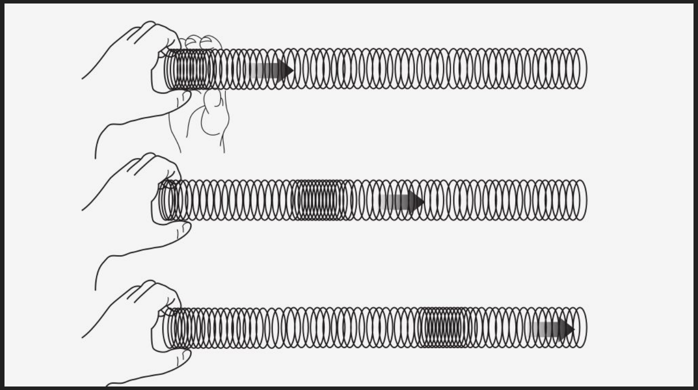
- The central questions in optics: Is light a localised particle (photon) or a non-localised wave?

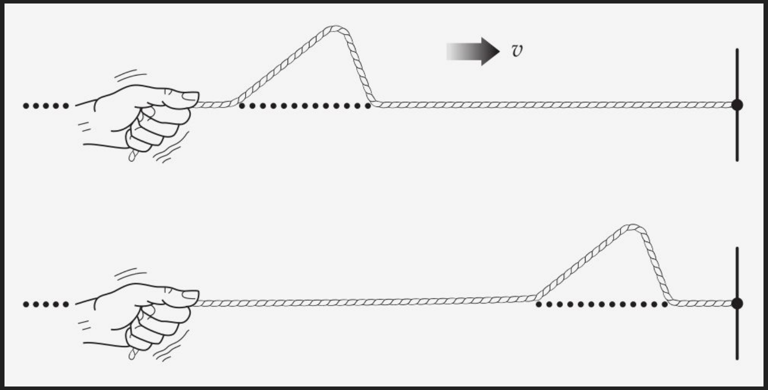


- Waves are continuous disturbances of matter or space that transport energy and momentum.

One-dimensional wave

- We are most familiar with mechanical waves, e.g. waves on strings, surface waves or sound waves.
- We distinguish between longitudinal waves (sound) and transverse waves (waves on strings).





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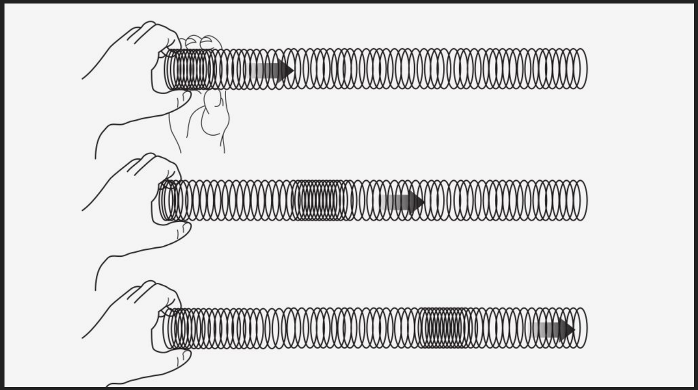
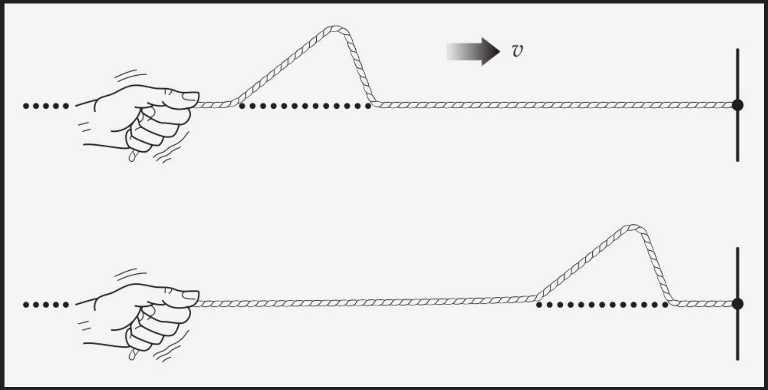
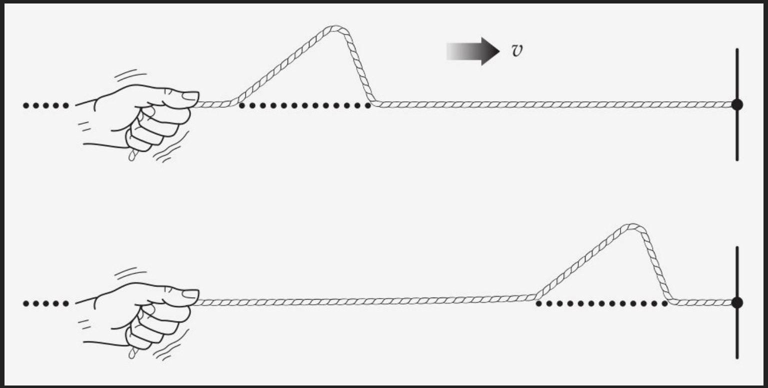




- The medium itself is not transported but the disturbance moves through the medium.

One-dimensional wave equation

- The disturbance is moving: its (constant) profile has to be a function of position and time, $\Psi(x,t)$.



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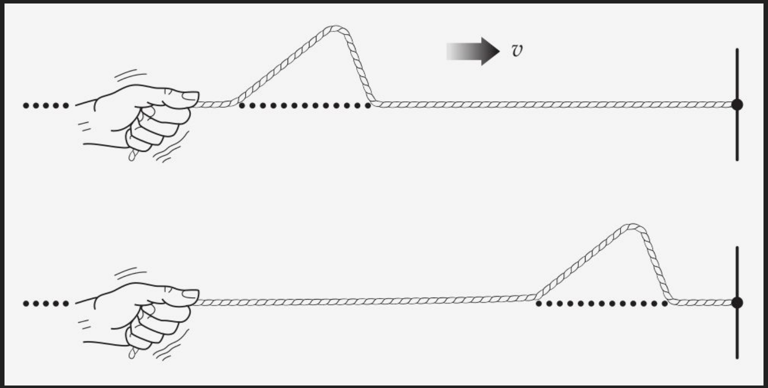

- Every (undamped) wave is fully characterised by a linear, homogeneous, second-order, partial differential equation, the differential wave equation:

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

(D'Alembert, 1747)

Harmonic waves

- The simplest waveform has a sine or cosine profile:

$$\psi(x,t)\big|_{t=0} = \psi(x) = A \sin kx$$

$$\psi(x,t) = A \sin k(x - vt)$$

- The maximum disturbance is called the wave amplitude A, while k is the propagation number.

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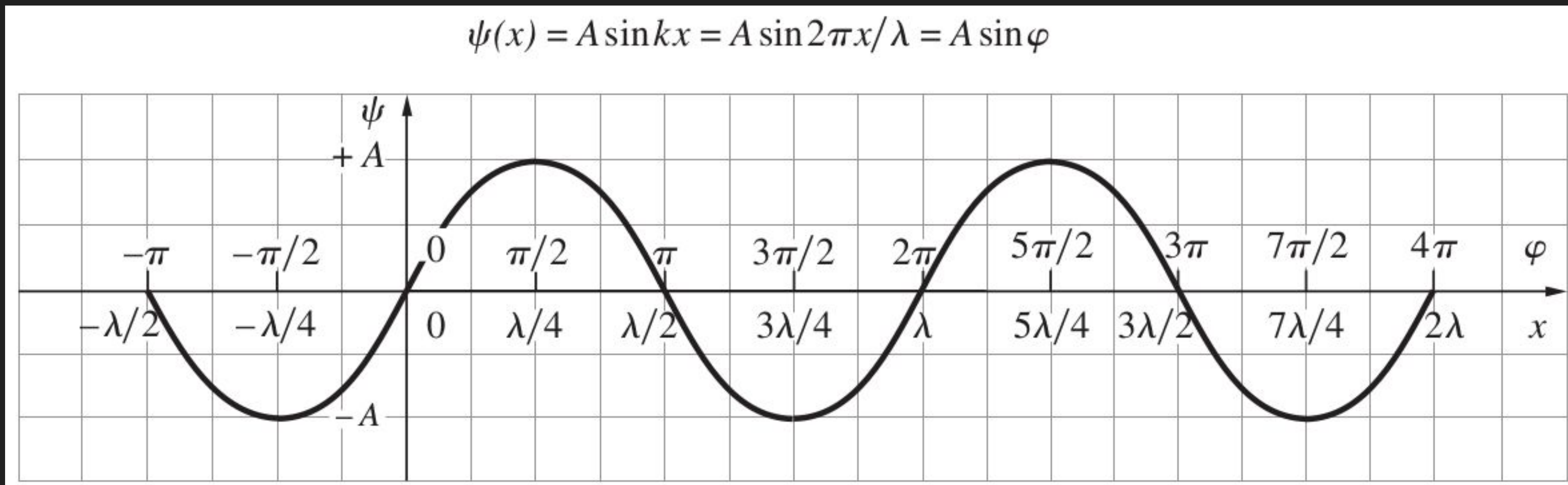
Exercise: Show that Ψ(x,t) solves the wave equation.

- The wave is periodic in space and time - the spatial period ƛ is known as the wavelength, related to k as

$$k = 2\pi/\lambda$$

Harmonic waves



- The sine argument is called the phase φ of the wave.
- We also define the temporal period τ, frequency ν, angular temporal frequency ω and wave number κ:

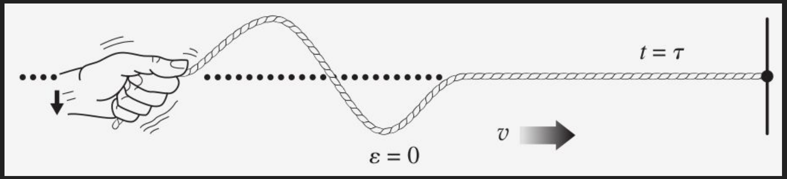
$$\tau = \lambda / v \qquad \nu \equiv 1/\tau \qquad \omega \equiv 2\pi/\tau = 2\pi\nu \qquad \kappa \equiv 1/\lambda$$

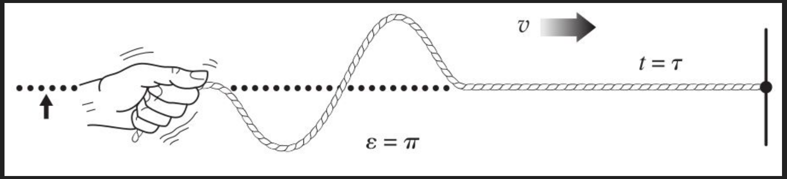
Phase and phase velocity

- In the most general form:

$$\psi(x, t) = A \sin(kx - \omega t + \varepsilon)$$

with the initial phase ε.





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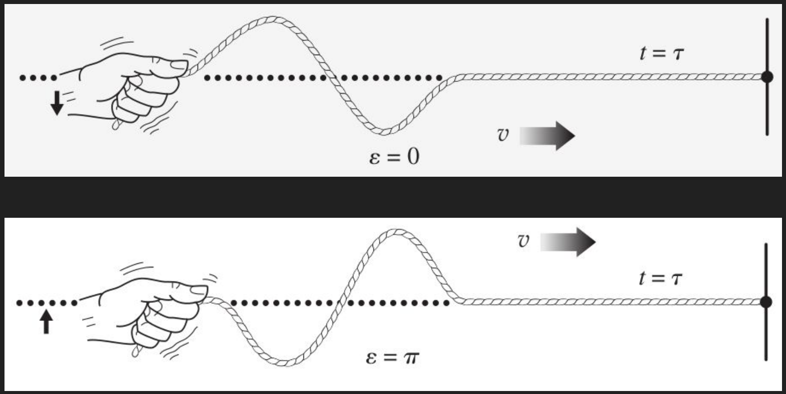


- The rate-of-change of phase with time and distance allows us to define the phase velocity:

$$\left|\left(\frac{\partial\varphi}{\partial x}\right)_t\right| = k \qquad \left|\left(\frac{\partial\varphi}{\partial t}\right)_x\right| = \omega \qquad \left(\frac{\partial x}{\partial t}\right)_\varphi = \pm\frac{\omega}{k} = \pm v$$

Superposition principle

- If Ψ_1 and Ψ_2 are solutions to the wave equation then $\Psi_1 + \Psi_2$ is also a solution, because

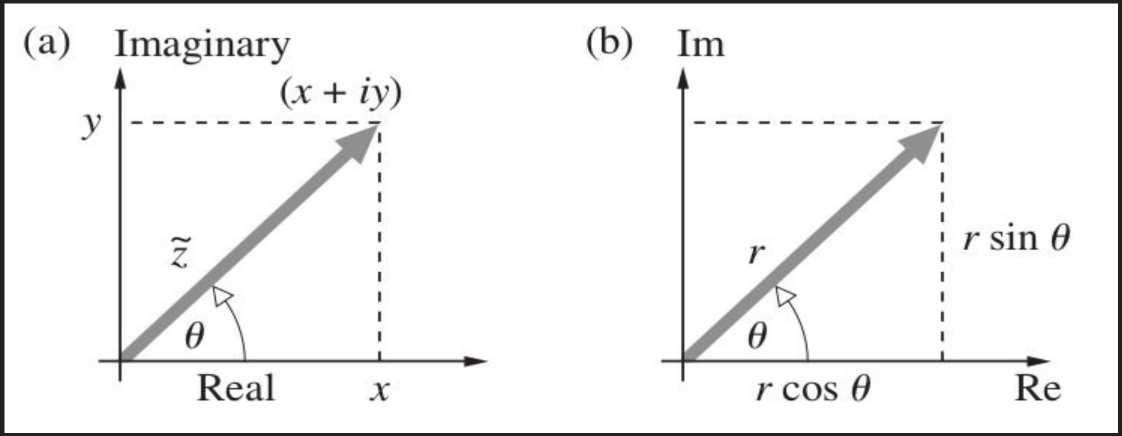
$$\frac{\partial^2 \psi_1}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi_1}{\partial t^2} \quad + \quad \frac{\partial^2 \psi_2}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi_2}{\partial t^2}$$

$$\frac{\partial^2}{\partial x^2}(\psi_1 + \psi_2) = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}(\psi_1 + \psi_2)$$

- The combined disturbance at one point is the algebraic sum of the individual constituent waves at that location. This will be crucial for wave interference.

Complex representation

- Dealing with trigonometric functions, it is convenient to use the complex-number representation



$$\tilde{z} = x + iy$$

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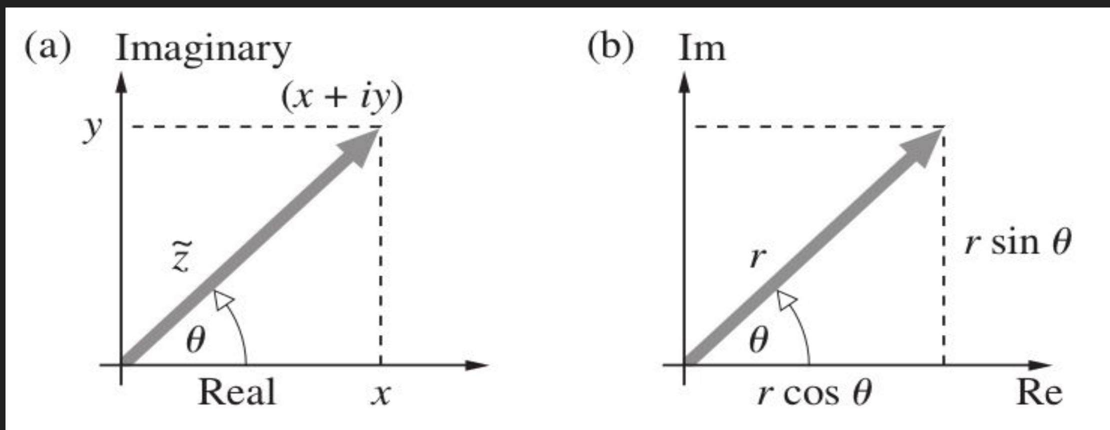

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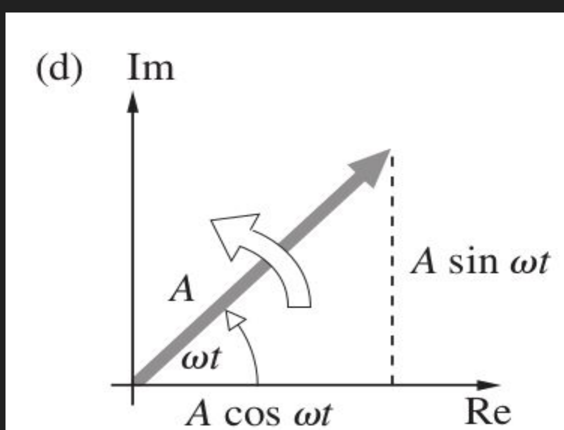

$$\psi(x, t) = Ae^{i(\omega t - kx + \varepsilon)} = Ae^{i\varphi}$$

- The (physical) wave corresponds to the real part of Ψ.

Plane waves

- Another way to illustrate disturbances is by looking at points of equal amplitude, occurring at the same φ.
    - The 3D surface of constant phase is called a wavefront.

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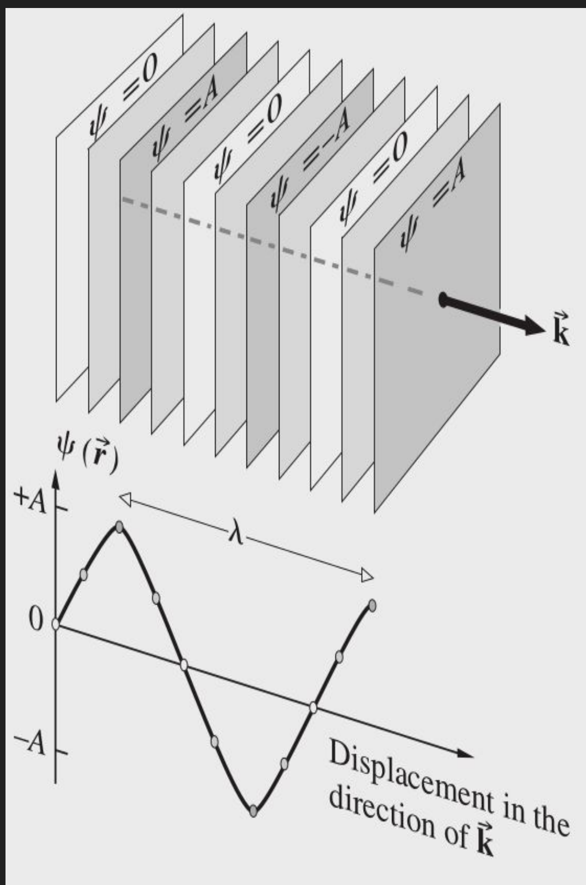

- The 3D surface of constant phase is called a wavefront.
- The simplest example of a 3D wave is the plane wave with

$$\psi(\vec{r}) = Ae^{i\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = \text{constant}$$

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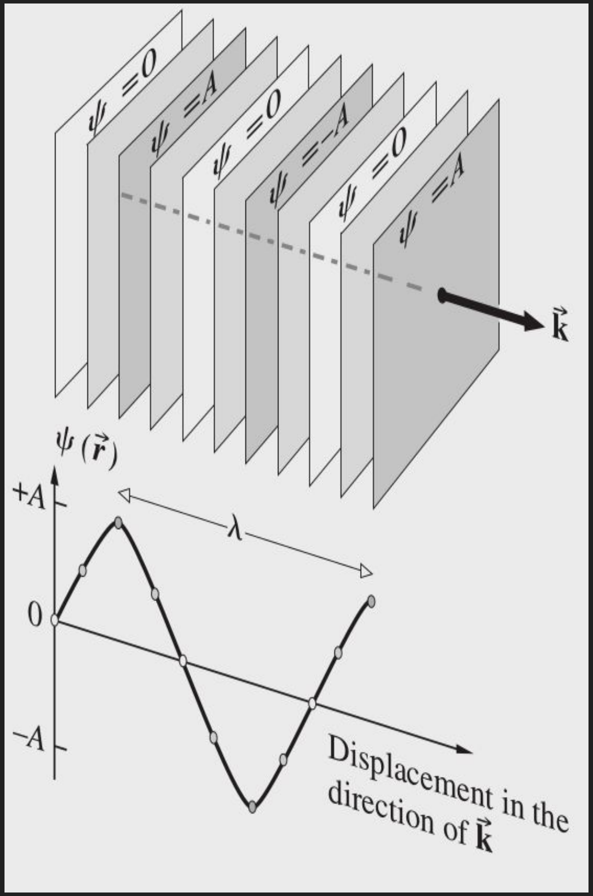


- The 3D surface of constant phase is called a wavefront.

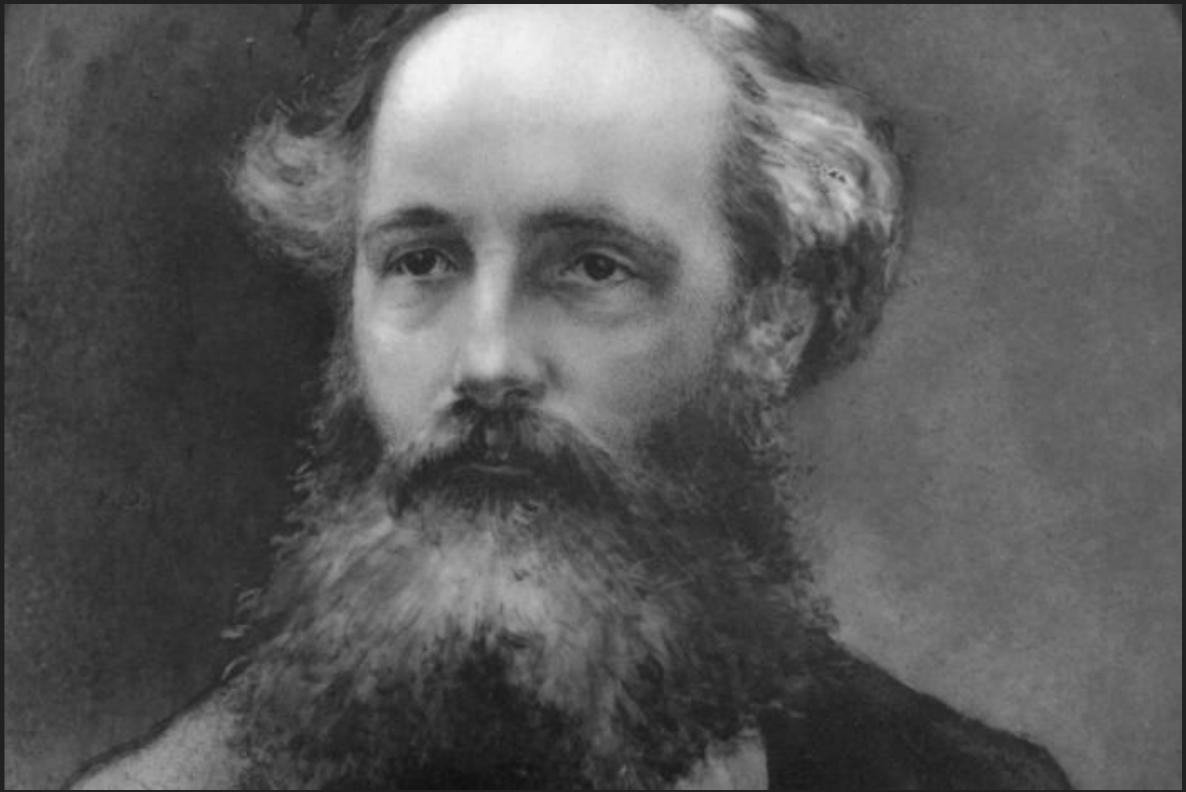
- The simplest example of a 3D wave is the plane wave with

$$\psi(\vec{r}) = Ae^{i\vec{k}\cdot\vec{r}} \qquad \vec{k}\cdot\vec{r} = \text{constant}$$

- Any 3D wave can be written as a combination of plane waves.

PHYS 434 Optics

Lecture 1: Course Introduction, Waves and **Electromagnetism** in a Nutshell



Light and Electromagnetism

- Work by Maxwell (and many others) showed that light is of electromagnetic nature and on macroscopic scales represented by a continuous wave.

- For many applications, it is sufficient to neglect the underlying quantum (particle) nature of light.

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- Work by Maxwell (and many others) showed that light is of electromagnetic nature and on macroscopic scales represented by a continuous wave.

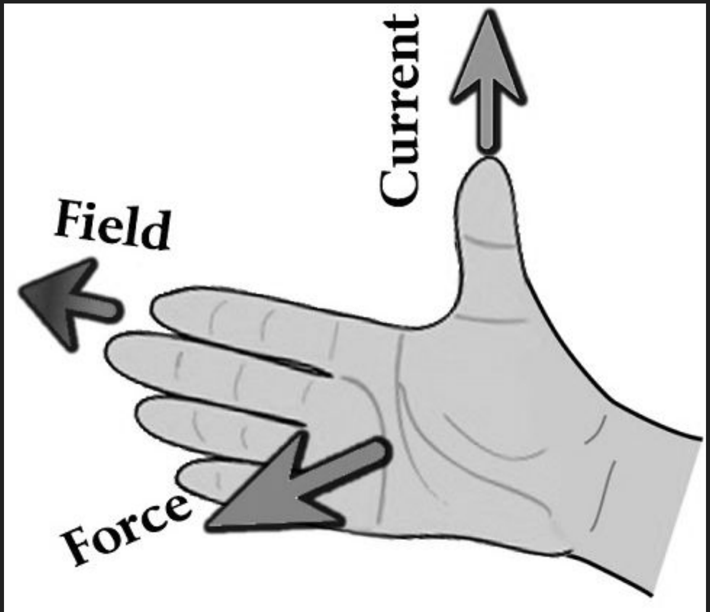
- For many applications, it is sufficient to neglect the underlying quantum (particle) nature of light.

- We will focus on this wave nature but have to keep in mind that there are situations, where this representation is completely inadequate, e.g. when we talk about the generation of light or its absorption.

Important quantities

- If a point charge q (moving with velocity $\vec{v}$) is immersed in an electric field $\vec{E}$ and a magnetic field $\vec{B}$, it experiences the Lorentz force

$$\vec{\mathbf{F}} = q\,\vec{\mathbf{E}} + q\,\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$



- Electric and magnetic fields depend on each other since a time-varying electric field $\vec{E}$ can generate a magnetic field $\vec{B}$ and vice versa.

Faraday's law

- Faraday was the first to discover that a changing magnetic field generated an electric current.
- By analysing the voltage (or electromotive force) induced in wire loops exposed to a magnetic field, he discovered that

Magnetic flux Φ_M

$$\oint_C \vec{\mathbf{E}} \cdot d\vec{\ell} = -\frac{d}{dt}\iint_A \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = -\frac{d\Phi_M}{dt}$$

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- Lenz's law: the induced voltage creates an induced field that opposes the flux change that caused it.

Gauss's law - electric

- It relates the flux of the electric field and sources of that flux, i.e. charges.
- If no sources or sinks exist within a region encompassed by a closed surface, the net flux through the surface equals zero.

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- If no sources or sinks exist within a region encompassed by a closed surface, the net flux through the surface equals zero.
- If charges (or a charge distribution ρ) are present:

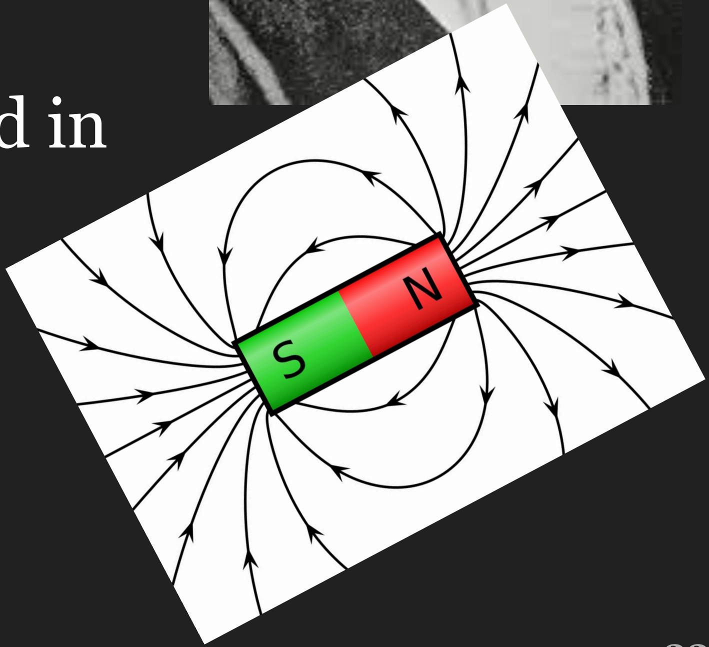
$$\Phi_E = \oiint_A \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \frac{1}{\epsilon_0} \sum q = \frac{1}{\epsilon_0} \iiint_V \rho \, dV$$

Vacuum permittivity ϵ_o

Gauss's law - magnetic

- There is no known magnetic counterpart to the electric charge, i.e. no magnetic monopoles.
- Magnetic fields do not converge toward some kind of magnetic charge but are instead described in terms of current distributions:

$$\Phi_M = \oiint_A \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = 0$$

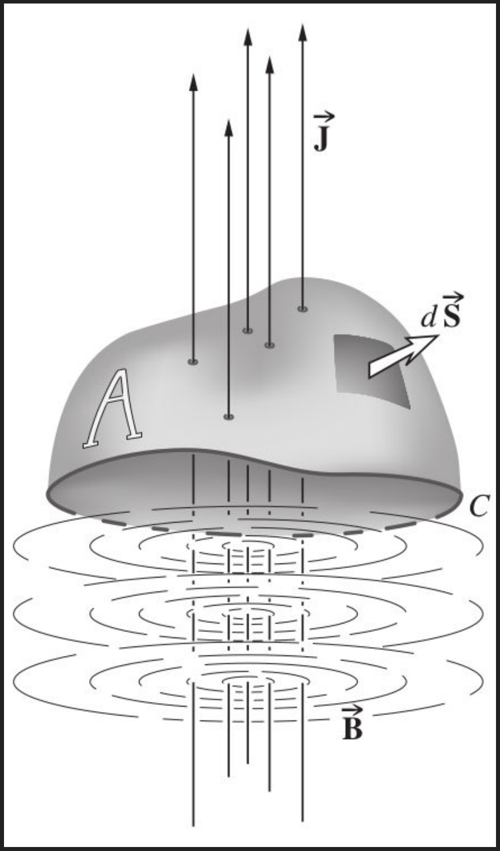


Ampère's law

- It relates the integrated magnetic field around a closed loop to the electric currents i (or a current density $\vec{J}$) passing through the loop:

Vacuum permeability μ_o

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 \sum i = \mu_0 \iint_A \vec{\mathbf{J}} \cdot d\vec{\mathbf{S}}$$



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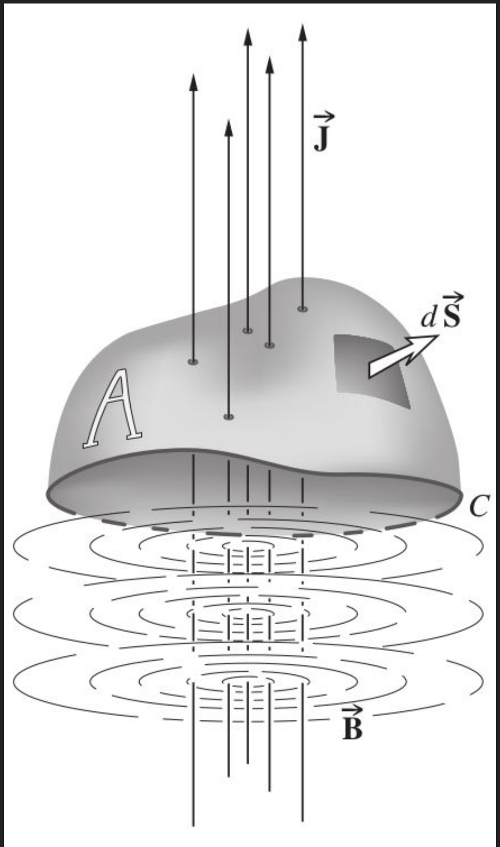


- $\vec{B}$ is not only created by q but also changes in $\vec{E}$. Accounting for the displacement current density:

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu \iint_A \left(\vec{\mathbf{J}} + \epsilon \frac{\partial \vec{\mathbf{E}}}{\partial t} \right) \cdot d\vec{\mathbf{S}}$$

Medium permittivity ϵ, medium permeability μ

Maxwell's equations I

- The four expressions derived from experimental observations are known as Maxwell's equations.
- In vacuum, where charges and currents are absent, Maxwell's equations read

$$\oint_C \vec{\mathbf{E}} \cdot d\vec{\ell} = -\iint_A \frac{\partial \vec{\mathbf{B}}}{\partial t} \cdot d\vec{\mathbf{S}} \qquad \oiint_A \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = 0$$

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 \epsilon_0 \iint_A \frac{\partial \vec{\mathbf{E}}}{\partial t} \cdot d\vec{\mathbf{S}} \qquad \oiint_A \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = 0$$

Maxwell's equations II

- Applying the divergence and Stokes' theorem, the integral equations can be rewritten in differential form.
- The full equations (in a medium) read

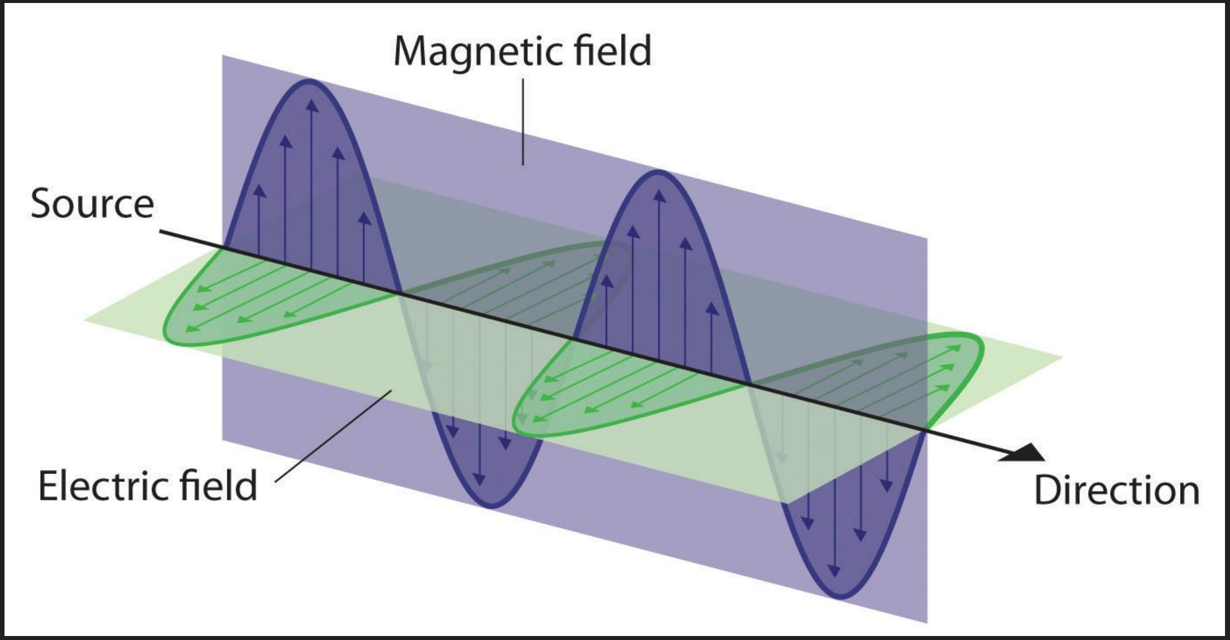
$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t} \qquad \nabla \cdot \vec{\mathbf{B}} = 0$$

$$\nabla \times \vec{\mathbf{B}} = \mu \left(\vec{\mathbf{J}} + \epsilon \frac{\partial \vec{\mathbf{E}}}{\partial t} \right) \qquad \nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon}$$

PHYS 434 Optics

Lecture 2: **Light Propagation, Lorentz Model** and Scattering

Reading: **3.2, 3.3.1, 3.3.2, 3.5**, 4.1, 4.2



Summary Lecture 1

- On macroscopic scales, light appears to be of electro-magnetic, wave-like nature.
- Maxwell's equations were deduced from experimental observations. In differential form, they read

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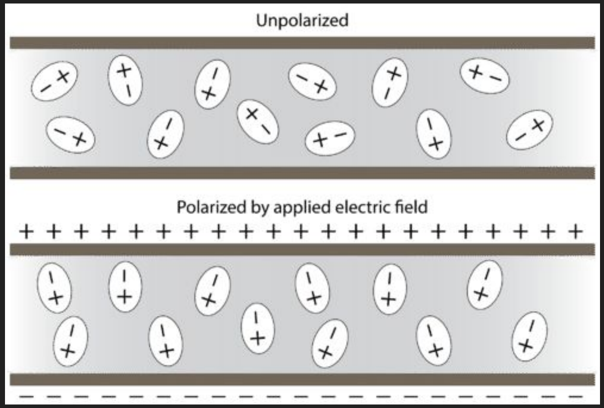
$$\nabla \times \vec{\mathbf{B}} = \mu \left(\vec{\mathbf{J}} + \epsilon \frac{\partial \vec{\mathbf{E}}}{\partial t} \right) \qquad \nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon}$$

- How does the medium actually affect these relations?

Electric polarisation

- When a dielectric is subjected to an applied electric field, the internal charge distribution is distorted.
- Positive and negative charges are separated, each pair forming a little dipole, which contributes to the internal field.



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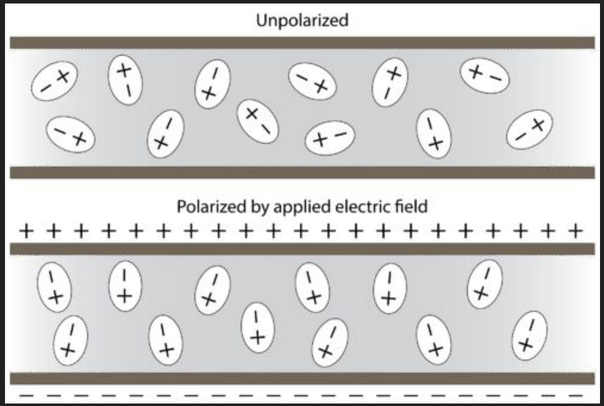

- The resulting dipole moment per unit volume is the electric polarisation $\vec{P}$, typically proportional to $\vec{E}$:

$$(\epsilon - \epsilon_0)\vec{\mathbf{E}} = \vec{\mathbf{P}}$$

Displacement field

- $\vec{P}$ measures the difference in $\vec{E}$ with/without medium.
- For convenience, the field alteration is often included by introducing the displacement field $\vec{D}$ via

$$\vec{\mathbf{D}} = \epsilon_0 \vec{\mathbf{E}} + \vec{\mathbf{P}} = \epsilon \vec{\mathbf{E}}$$

the first constitutive equation in electromagnetism.

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  the first constitutive equation in electromagnetism.
- Using this relation, we can rewrite Gauss's law as

$$\nabla \cdot \vec{\mathbf{D}} = \rho$$

- $\vec{\mathrm{D}}$ is determined by the distribution ϱ of free charges.

Magnetisation

- Similarly, when a magnetic medium is exposed to a magnetic field is becomes magnetically polarised.
- The quantity capturing the presence of permanent and induced magnetic moments is the magnetic polarization or magnetization vector $\vec{M}$.

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- The quantity capturing the presence of permanent and induced magnetic moments is the magnetic polarization or magnetization vector $\vec{M}$.
- $\vec{M}$ represents the total magnetic dipole moment per unit volume and is, thus, analogous to the electric polarisation field $\vec{P}$. For linear media, we have

$$\vec{\mathbf{M}} = \mu_0^{-1}\vec{\mathbf{B}} - \mu^{-1}\vec{\mathbf{B}}$$

Magnetic field

- To incorporate the influence of a magnetically polarised medium, we introduce the auxiliary field $\vec{\mathrm{H}}$ via

$$\vec{\mathbf{H}} = \mu_0^{-1}\vec{\mathbf{B}} - \vec{\mathbf{M}} = \mu^{-1}\vec{\mathbf{B}}$$

  the second constitutive equation.

- Note that $\vec{\mathrm{H}}$ is usually called the magnetic field, while $\vec{\mathrm{B}}$ is referred to as the magnetic induction.

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- Note that $\vec{\mathrm{H}}$ is usually called the magnetic field, while $\vec{\mathrm{B}}$ is referred to as the magnetic induction.

- Ampère's law (controlled by free currents) now reads

$$\nabla \times \vec{\mathbf{H}} = \vec{\mathbf{J}} + \frac{\partial \vec{\mathbf{D}}}{\partial t}$$

Maxwell’s equation in matter

- The ‘microscopic’ version of Maxwell's equations determines the fields in terms of all the (possibly atomic-level) charges and currents presents.

Maxwell's equation in matter

- The ‘microscopic’ version of Maxwell's equations determines the fields in terms of all the (possibly atomic-level) charges and currents presents.
- The ‘macroscopic’ form absorbs bound charges and currents into $\vec{D}$, $\vec{H}$ and requires constitutive relations:

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t} \qquad \nabla \cdot \vec{\mathbf{B}} = 0$$

$$\nabla \times \vec{\mathbf{H}} = \vec{\mathbf{J}} + \frac{\partial \vec{\mathbf{D}}}{\partial t} \qquad \nabla \cdot \vec{\mathbf{D}} = \rho$$

Ohm's law

- Electromagnetism requires a third constitutive relation that connects the electric field and the current.

- Experiments suggest that in conductors, the electric field (and, hence, the force acting on each electron) controls the charge flow. Ohm's law thus reads:

$$\vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

- The proportionality constant σ is the conductivity.

EM wave equation I

- For uncharged, non-conducting matter, it is possible to combine Maxwell's equations to obtain these (equivalent) partial differential equations:

$$\nabla^2\vec{\mathbf{B}} - \mu\epsilon\frac{\partial^2\vec{\mathbf{B}}}{\partial t^2} = 0 \qquad \nabla^2\vec{\mathbf{H}} - \mu\epsilon\frac{\partial^2\vec{\mathbf{H}}}{\partial t^2} = 0$$

$$\nabla^2\vec{\mathbf{E}} - \mu\epsilon\frac{\partial^2\vec{\mathbf{E}}}{\partial t^2} = 0 \qquad \nabla^2\vec{\mathbf{D}} - \mu\epsilon\frac{\partial^2\vec{\mathbf{D}}}{\partial t^2} = 0$$

- This highlights the symmetry of Maxwell's equations, and the interdependence of the different fields.

EM wave equation II

- Exercise: Derive one of these wave equations for uncharged, non-conducting matter. Use

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t} \qquad \nabla \cdot \vec{\mathbf{B}} = 0$$

$$\nabla \times \vec{\mathbf{H}} = \vec{\mathbf{J}} + \frac{\partial \vec{\mathbf{D}}}{\partial t} \qquad \nabla \cdot \vec{\mathbf{D}} = \rho$$

$$\vec{\mathbf{H}} = \mu_0^{-1}\vec{\mathbf{B}} - \vec{\mathbf{M}} = \mu^{-1}\vec{\mathbf{B}}$$

$$\vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}$$

$$\nabla \times (\nabla \times\ ) = \nabla(\nabla \cdot) - \nabla^2$$

$$\vec{\mathbf{D}} = \epsilon_0\vec{\mathbf{E}} + \vec{\mathbf{P}} = \epsilon\vec{\mathbf{E}}$$

EM wave speed

- Each component of the fields E/B/D/H $_{x,y,z}$ obeys the scalar differential wave equation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

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- EM waves in vacuum move at the speed of light

$$v = 1/\sqrt{\epsilon_0\mu_0} = c = 2.997\,924\,58 \times 10^8 \text{ m/s}$$

- c is independent of the motion of the source and the observer, which is crucial for special relativity.

Electromagnetic waves I

- Gauss's laws ensure that plane waves have electric and magnetic fields that are perpendicular to the direction of propagation, i.e. they are transverse.
- Consider a plane wave moving in x-direction, i.e. $\vec{E}=\vec{E}(x,t)$. Gauss's law in free space now dictates:

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$$\frac{\partial E_x}{\partial x} = 0 \quad \rightarrow \quad \text{for a travelling wave:} \quad E_x = 0$$

- A EM wave has no field component in the direction of propagation, only perpendicular components $E_{y,z}$.

Electromagnetic waves II

- To fully characterise a wave, we need to specify the direction of $\vec{E}$, which is typically referred to as the polarisation. For a linearly-polarised wave, $\vec{E}$ is fixed.

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- To fully characterise a wave, we need to specify the direction of $\vec{E}$, which is typically referred to as the polarisation. For a linearly-polarised wave, $\vec{E}$ is fixed.
- Consider $\vec{E} = \hat{j} E_y(x,t)$. Faraday's law then dictates

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \quad \rightarrow \text{ only } \vec{B} = \hat{k} B_z(x,t)$$

- For the special case of a harmonic wave, we have

$$E_y(x, t) = E_{0y} \cos\left[\omega(t - x/c) + \varepsilon\right]$$

Electromagnetic waves III

- Integrating Faraday's law, the magnetic induction is

$$B_z(x, t) = -\int \frac{\partial E_y}{\partial x} dt = -\frac{E_{0y}\omega}{c} \int \sin[\omega(t - x/c) + \varepsilon] dt$$

$$= \frac{1}{c} E_{0y} \cos[\omega(t - x/c) + \varepsilon] = \frac{1}{c} E_y(x, t)$$

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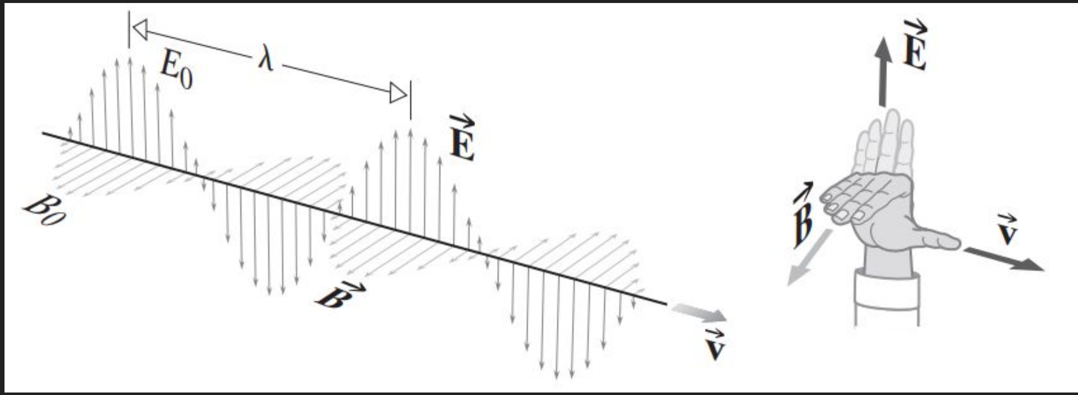
- Both fields have the same time-dependence and differ only by the scalar c, implying that they are in phase.
- We can use this information to construct a snapshot.

Electromagnetic waves IV



- $\vec{E}$- and $\vec{B}$-fields are two aspects of a single phenomenon, the EM field, generate by a moving charge.
- A disturbance in the field is a wave, moving beyond its source - the time-varying electric and magnetic fields regenerate each other in an endless cycle, e.g. light from stars takes millions of years to reach the Earth.

Poynting vector I

- In optics, we often do not deal with the vector quantities but with light intensity (energy density flux).
- The energy density u is carried equally between the magnetic and electric field and given by

$$u = u_E + u_B = \frac{\epsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2 = \epsilon_0 E^2 = \frac{1}{\mu_0}B^2$$

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- Intensity is given by magnitude of Poynting vector $\vec{S}$

$$\vec{\mathbf{S}} = \frac{1}{\mu_0} \vec{\mathbf{E}} \times \vec{\mathbf{B}}$$

Poynting vector II

- For the harmonic wave considered earlier, we obtain

$$\vec{\mathbf{S}} = c\,\epsilon_0\,E_{0y}^2\cos^2[\omega(t - x/c) + \varepsilon]\,\hat{\mathbf{i}}$$

- We expect a plane wave to have constant intensity, but this expression is clearly time-dependent. One has to average over many cycles to get the intensity.

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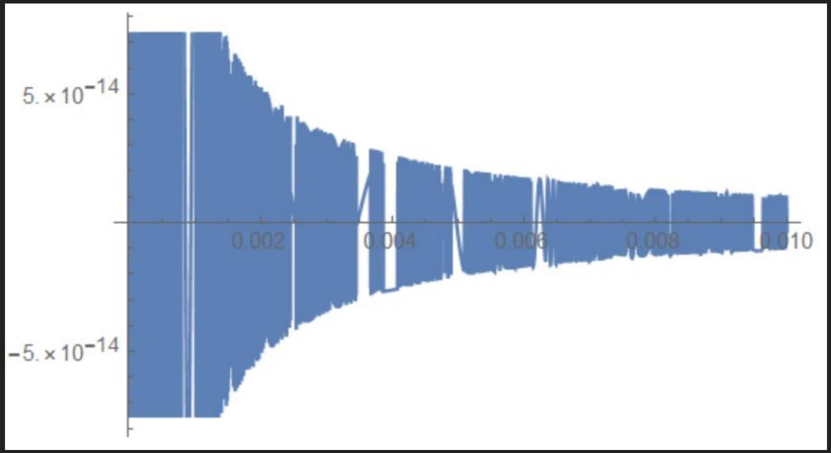
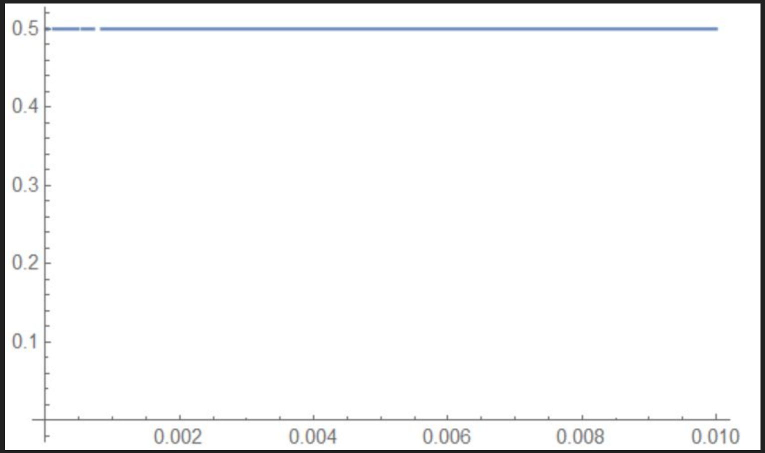
- We expect a plane wave to have constant intensity, but this expression is clearly time-dependent. One has to average over many cycles to get the intensity.
- Time-average of a function f(t) over an interval T is

$$\langle f(t)\rangle_{\mathrm{T}} = \frac{1}{T}\int_{t-T/2}^{t+T/2} f(t)\,dt$$

Averaging harmonic functions

- In the case of the harmonic wave, we have to average cos and $\cos^2$ to calculate the average of the EM fields and the Poynting flux, respectively.
- Perform the corresponding integrals to find that the former vanishes, while the latter is constant:





Irradiance

- In optics, the ‘amount’ of light illuminating a surface is referred to as irradiance I, which is the magnitude of the time-averaged Poynting flux.
- For a harmonic wave in vacuum, this leads to

$$I \equiv \langle S \rangle_{\mathrm{T}} = \frac{c\epsilon_0}{2} E_0^2$$

proportional to the square amplitude of the electric field. Or generally for a linear, isotropic medium

$$I = \epsilon v \langle E^2 \rangle_{\mathrm{T}}$$

Light in bulk matter

- The response of non-conducting materials, e.g. air, lenses, prisms, etc. to EM fields is crucial in Optics.
- Introducing a homogeneous, isotropic dielectric changes ϵ_0 to ϵ and μ_0 to μ, and thus the phase speed:

$$v = 1/\sqrt{\epsilon\mu}$$

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- Introducing a homogeneous, isotropic dielectric changes ϵ_0 to ϵ and μ_0 to μ, and thus the phase speed:

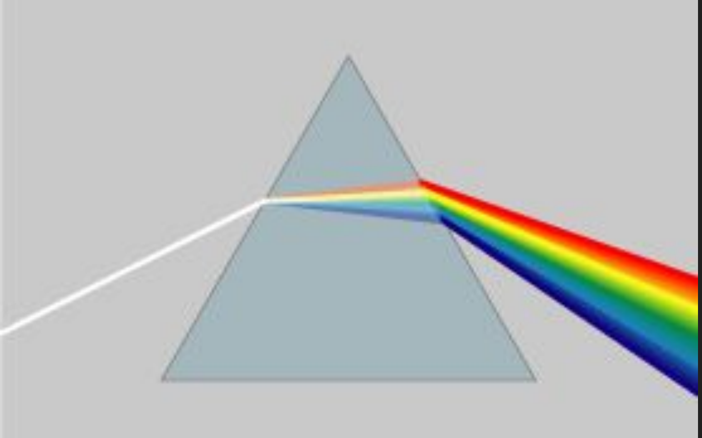
$$v = 1/\sqrt{\epsilon\mu}$$

- The ratio of the wave speed in vacuum to that in matter is called the absolute index of refraction n:

$$n \equiv \frac{c}{v} = \pm\sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}}$$

Index of refraction

- In some situations it is possible to characterise a medium by a constant n, e.g. for water n=1.33.
- In general, n depends on frequency: As discovered by Newton, different colours of light travel at different phase velocities in the same material: dispersion.



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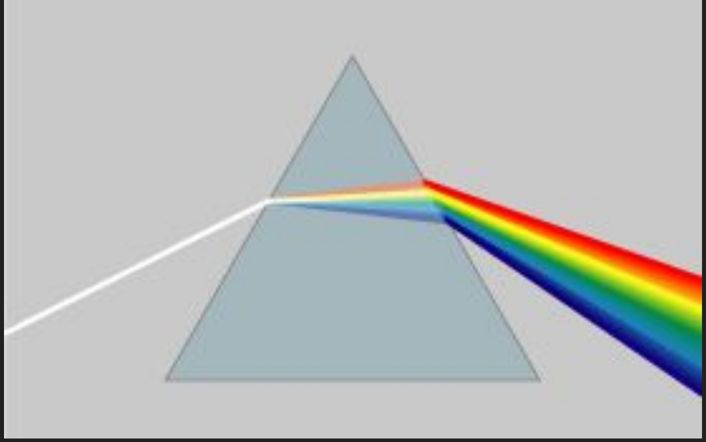
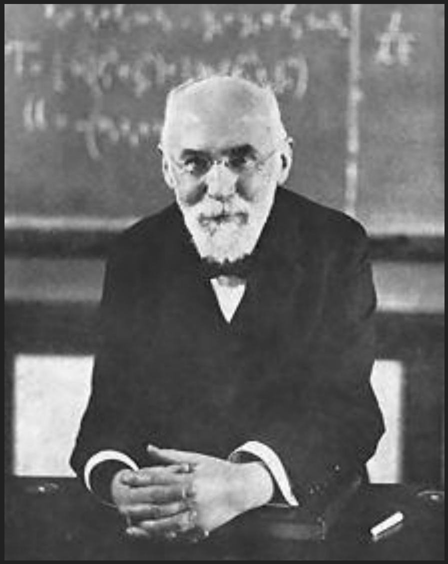
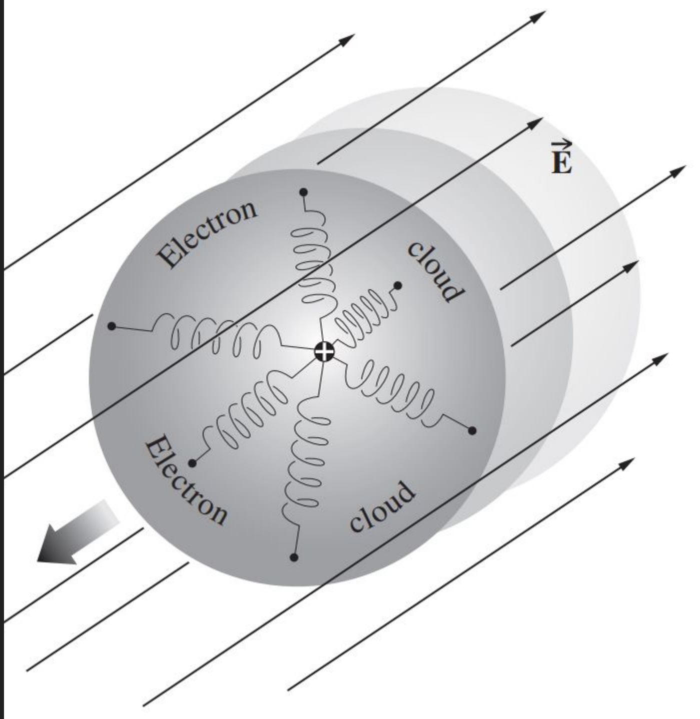
- In general, n depends on frequency: As discovered by Newton, different colours of light travel at different phase velocities in the same material: dispersion.
- Typically, n is also complex. The imaginary contribution leads to the exponential attenuation of light: absorption.



Lorentz model I



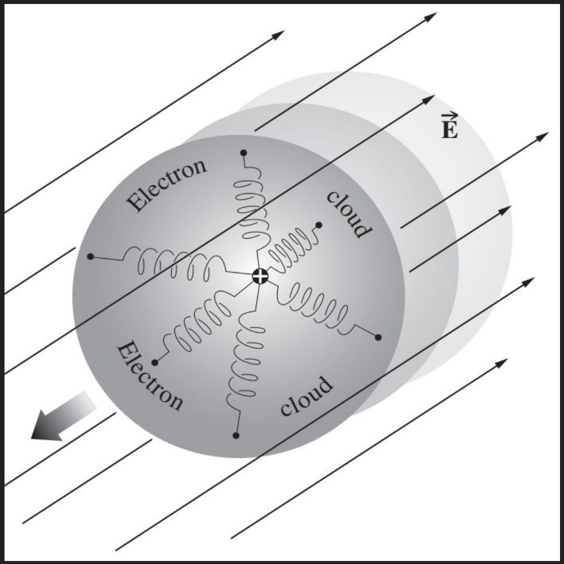
- To understand the features of n, we look at the microphysics of how a dielectric medium responds to light.



- In the Lorentz model, a material is decomposed into atoms consisting of electrons (or an rather electron cloud) that are bound to a fixed ionic core by springs.

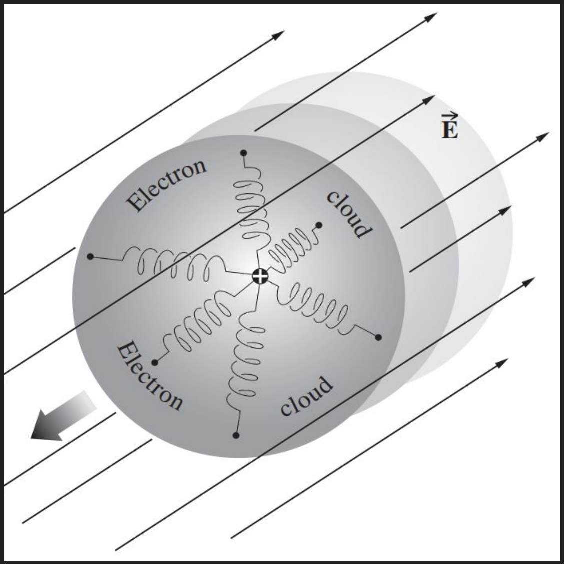
Lorentz model II

- Displacing the electrons results in a polarisation, which eventually determines the index of refraction.



- The EM field of an incident wave will drive the electron cloud into oscillation, i.e. the cloud vibrates at the frequency of the incident light.

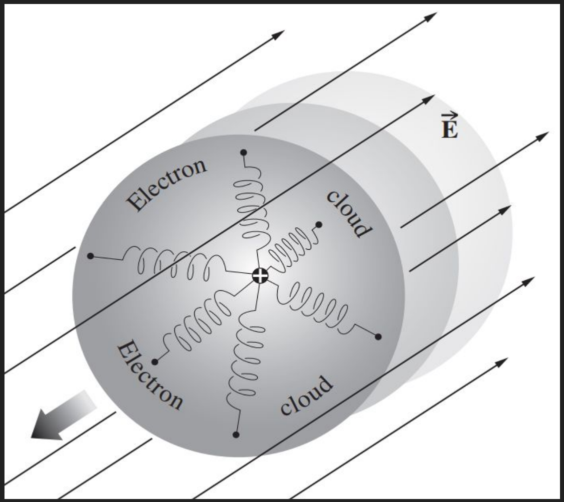
Lorentz model II



- Displacing the electrons results in a polarisation, which eventually determines the index of refraction.
- The EM field of an incident wave will drive the electron cloud into oscillation, i.e. the cloud vibrates at the frequency of the incident light.
- Each atom itself, thus, acts as an oscillating dipole and will radiate at that same frequency. The superposition of all resulting waves reflects the full behaviour of the isotropic dielectric medium.

Lorentz model III

- Mathematically, this is described by a forced harmonic oscillator, since an electron driven from its equilibrium position oscillates at a characteristic frequency ω_o



$$\omega_0 = \sqrt{k_E/m_e}$$

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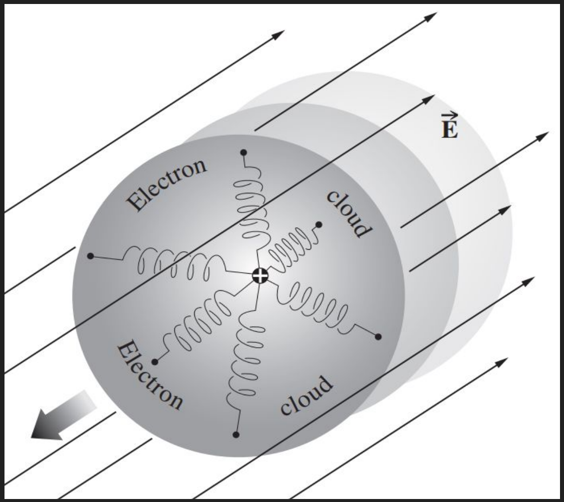

$$\omega_0 = \sqrt{k_E/m_e}$$

- For a harmonic driving force, Newton's second law gives the equation of motion (i.e. a differential equation for the displacement). Without damping, it reads

$$q_e E_0 \cos \omega t - m_e \omega_0^2 x = m_e \frac{d^2 x}{dt^2}$$

Lorentz model IV

- In the long-term limit, the electron will oscillate with the forcing electric field. This provides an ansatz for the time-dependence and hence

$$x(t) = x_0 \cos \omega t \quad \rightarrow \quad x(t) = \frac{q_e/m_e}{(\omega_0^2 - \omega^2)} E_0 \cos \omega t$$

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- After calculating the polarisation, we can determine the index of refraction (see PS #1 for more details):

$$n^2(\omega) = 1 + \frac{N q_e^2}{\epsilon_0 m_e} \left(\frac{1}{\omega_0^2 - \omega^2} \right)$$

PHYS 434 Optics

Lecture 2: Light Propagation, Lorentz Model and Scattering

Reading: 3.2, 3.3.1, 3.3.2, 3.5, 4.1, 4.2

Propagation of light

- Most of you will have dealt with the properties of light from a macroscopic perspective, e.g. the law of refraction, a view that can be very misleading.
- We now have the tools to understand how light interacts with bulk matter and explain its behaviour.

Propagation of light

- Most of you will have dealt with the properties of light from a macroscopic perspective, e.g. the law of refraction, a view that can be very misleading.
- We now have the tools to understand how light interacts with bulk matter and explain its behaviour.
- The processes of transmission, reflection, and refraction are all macroscopic manifestations of scattering on microscopic levels, which represents the absorption and prompt re-emission of electro-magnetic radiation by electrons.

Scattering

- Light propagating through empty space continues indefinitely, as no scattering takes place. If however, light moves through, e.g. air things are different.
- Each molecule acts like a little oscillator, which re-emits radiation of the same frequency, i.e. a small fraction of incident light is elastically scattered.

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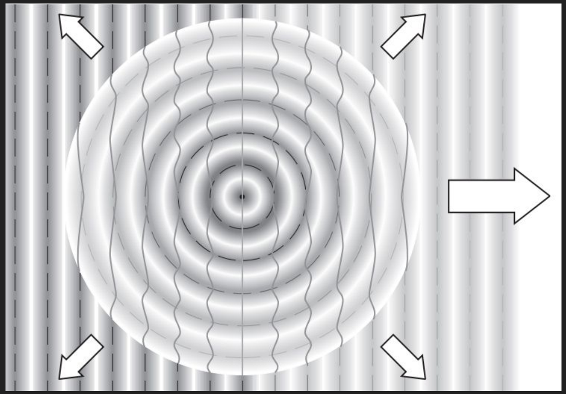
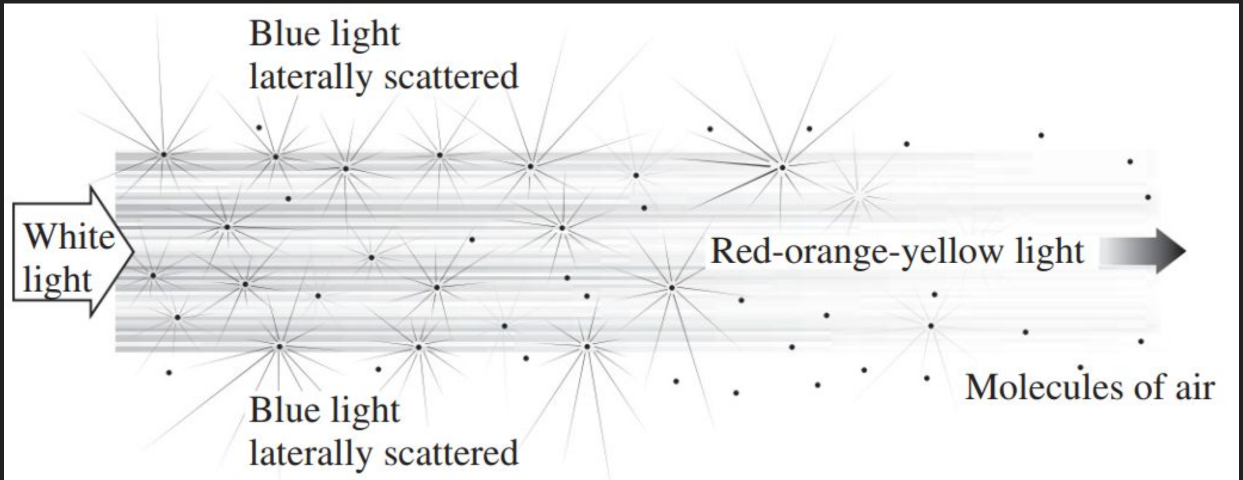


- Molecules are randomly oriented, so the light emanates in every direction: incoming plane wave is scattered into a spherical wave.

Rayleigh scattering

- The intensity of scattered light increases with frequency, because molecules have resonances in the UV: blue light will be scattered more than red light.
- For scatterers with a size $\ll \lambda$, the intensity goes as ν^4.



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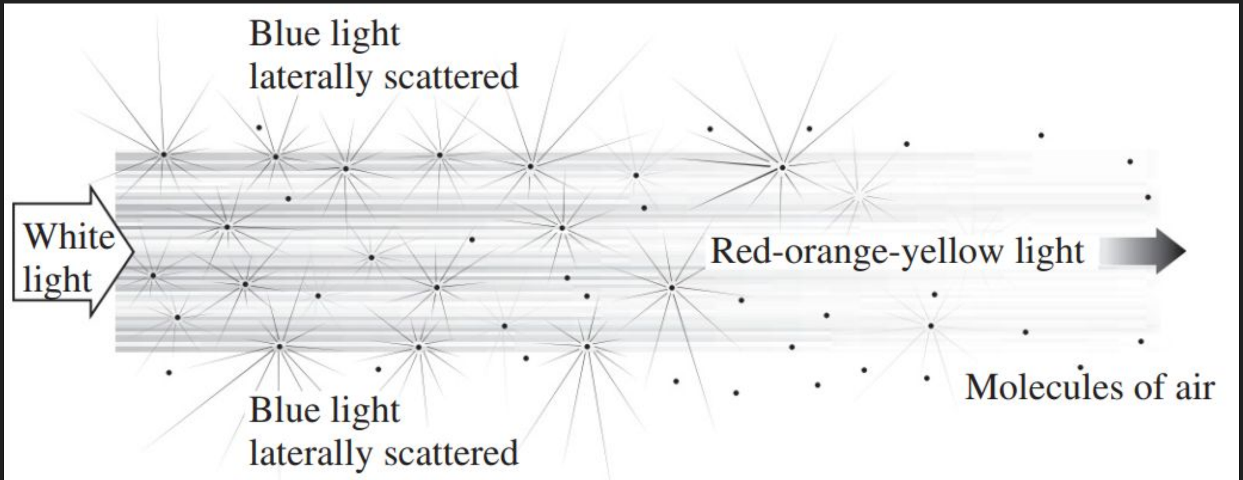
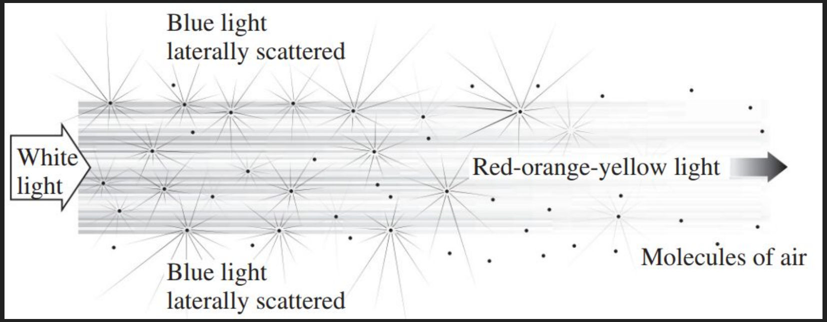


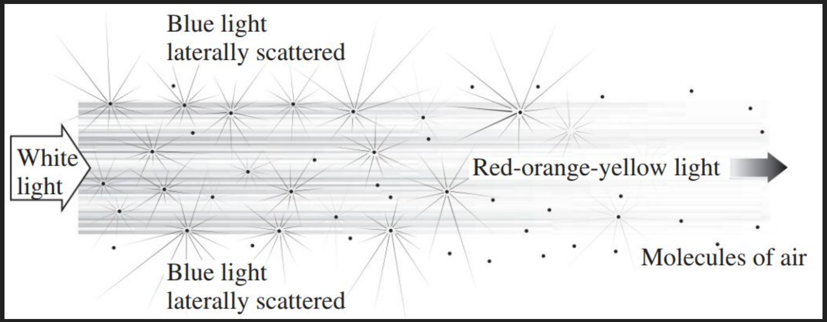
- This is the reason that, e.g. the sky looks blue to us.

Scattering and interference



- The e.g. outer atmosphere is a dilute medium and laterally scattered waves are independent of each other.
- They do not have a well-defined phase relationship and no interference (additive wave superposition) takes place in lateral directions.

Scattering and interference



- The e.g. outer atmosphere is a dilute medium and laterally scattered waves are independent of each other.
- They do not have a well-defined phase relationship and no interference (additive wave superposition) takes place in lateral directions.
- However, the light emitted in forward direction is phase-connected and can interfere constructively.

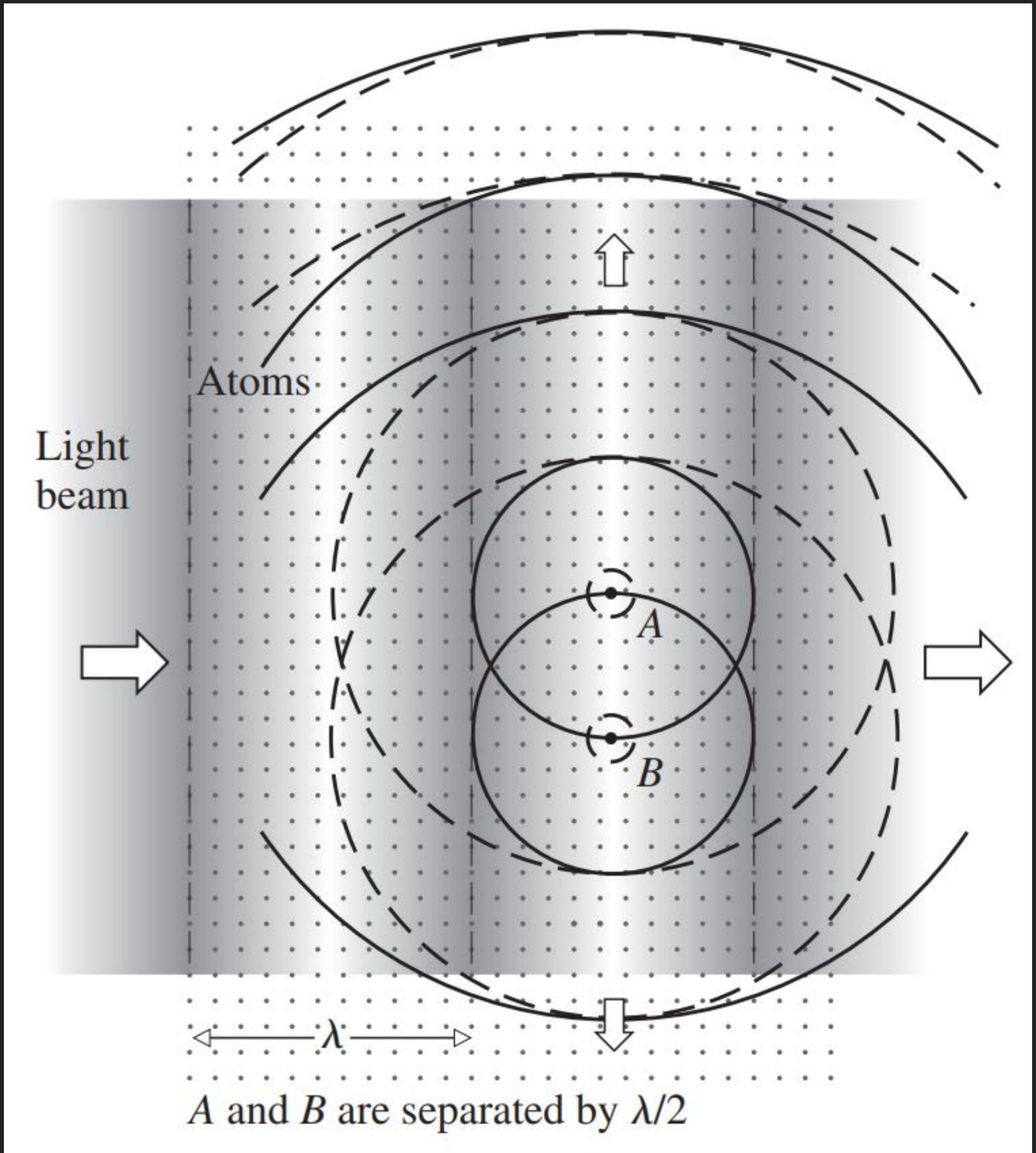
Light in a dense medium I

- This is not what we observe when shining light through a lense, water, etc. - what is different?

Light in a dense medium I

- This is not what we observe when shining light through a lense, water, etc. - what is different?
- A dense medium contains many scatterers per wave-length and while constructive interference persists in forward direction, waves will interfere destruc-tively in all other directions.
- The denser / more homogeneous (ordered) the sub-stance through which light moves, the better the des-tructive interference and less the lateral scattering.

Light in a dense medium II



- Interference redis-tributes energy from regions where it is destructive to those where it is constructive.

Microscopic view of refraction I

- We can use these ideas to understand refraction:
- An incident wave polarises the atoms, which start to oscillate and emit spherical waves (moving at c).
- Scattered waves interfere to produces a secondary wave (again moving at c) in forward direction, which however has a different phase to the incident one.

Microscopic view of refraction I

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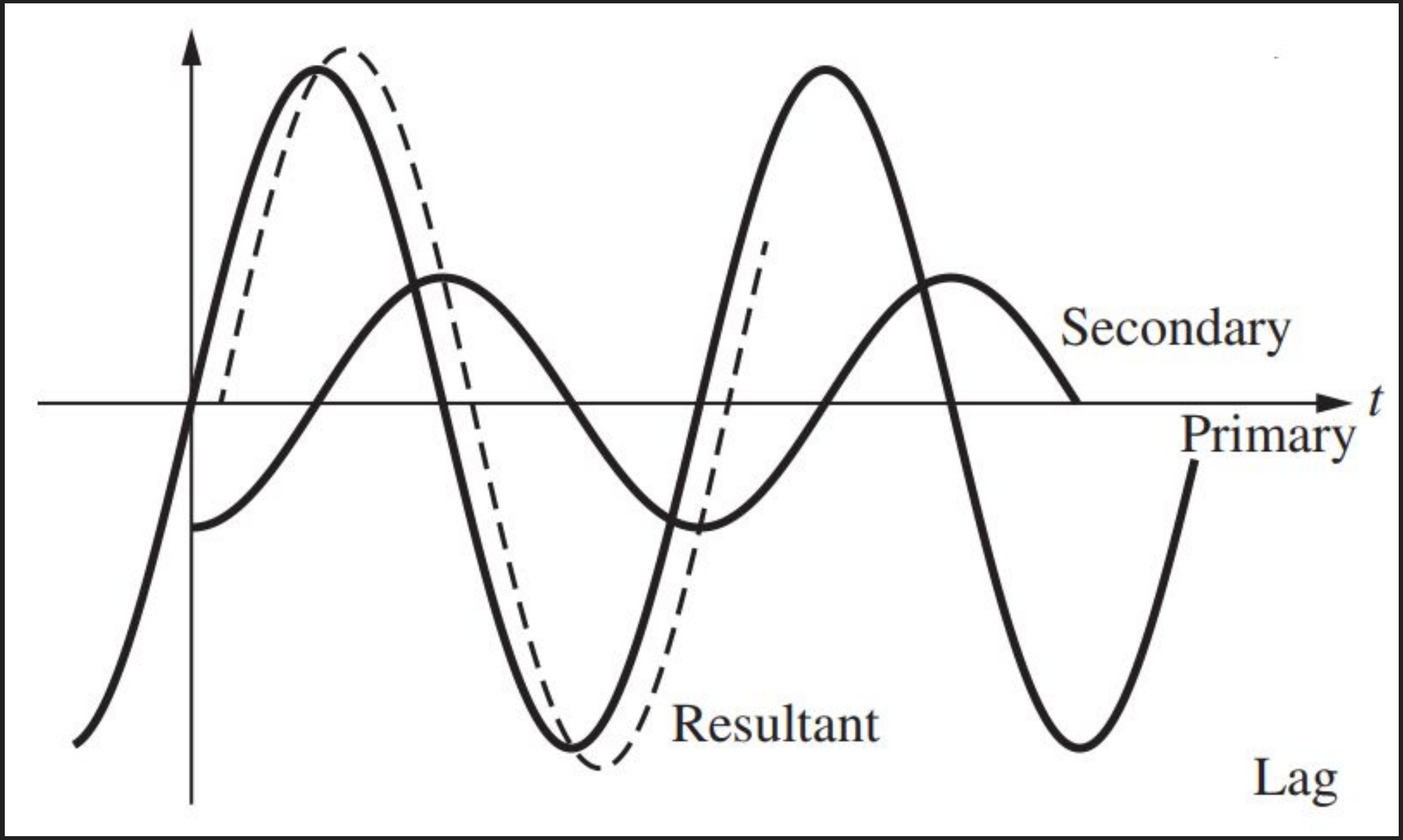
- Scattered waves interfere to produces a secondary wave (again moving at c) in forward direction, which however has a different phase to the incident one.
- The secondary wave interferes with the primary wave and introduces a phase shift into field, which appears as a shift in the apparent phase velocity of the transmitted beam from its nominal value of c.

Microscopic view of refraction II

- We can illustrate this in the following way:



Summary Lecture 2

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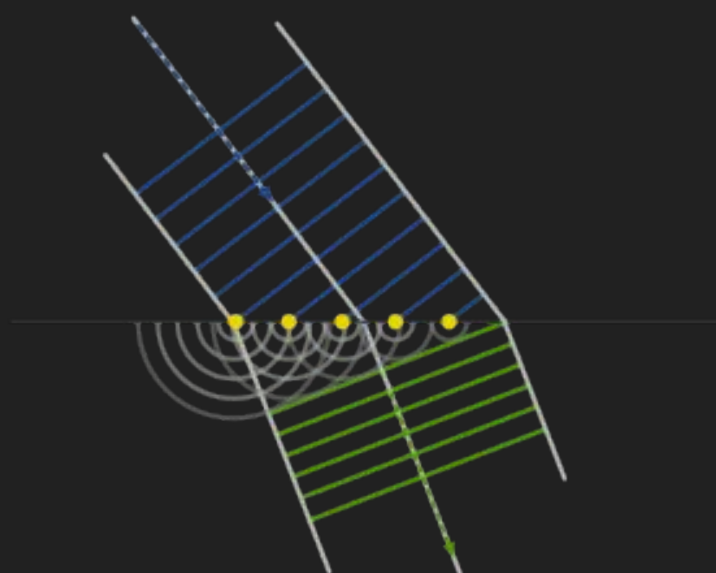
Summary Lecture 2

- Maxwell's equations capture the physics of the EM fields, whose disturbances are transverse waves that travel with the constant speed of light in vacuum.
- EM waves travel with a slower speed in matter, which is captured by a frequency-dependent, complex index of refraction. Microscopically this can be explained via the Lorentz model (many driven oscillators).
- Refraction can be understood on a microscopic level as the repeated elastic scattering of EM waves, which interfere to alter the apparent phase velocity.

PHYS 434 Optics

Lecture 3: Reflection, Refraction, Huygens' and Fermat's Principle

Reading: 4.3 - 4.5, 5.5.1



Admin

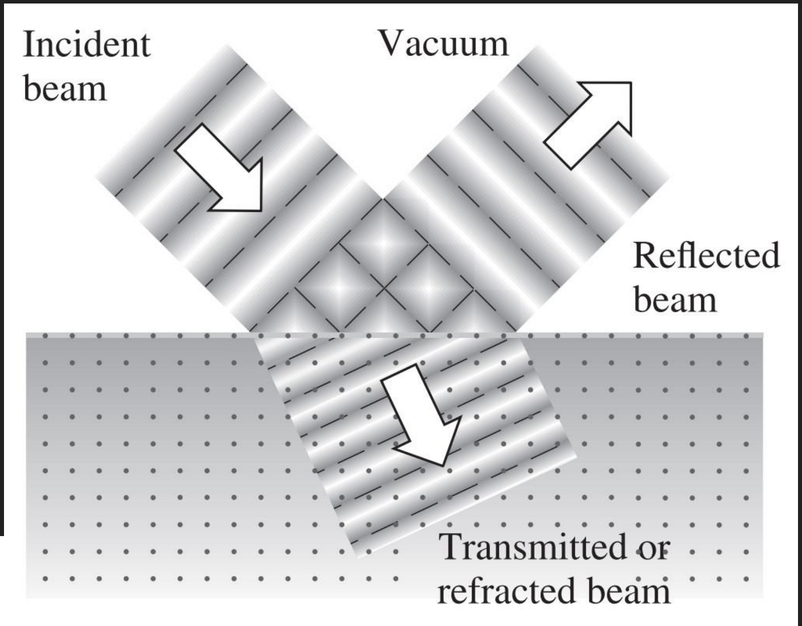
- Results of doodle poll:
  - Instructor office hours: Monday, 11am-12pm
  - TA office hours: Thursday, 4pm - 5pm
  - Weekly tutorial, Monday, 4pm - 5pm
    in the MSI conference room, 3550 Rue University

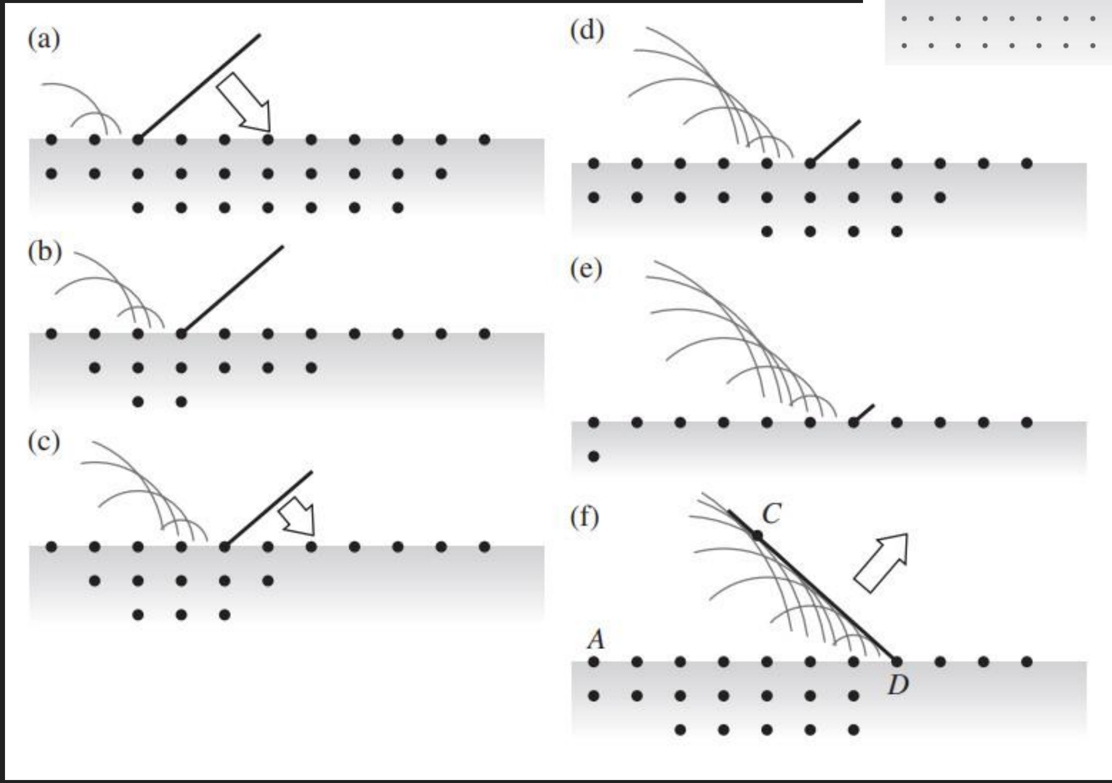
- First problem set is available on myCourses website:
  - Grader: Ziggy
  - Due date: Monday, Jan 21 (beginning of class)
  - Submit standard problems on paper, while uploading computational ones online

Summary Lecture 2

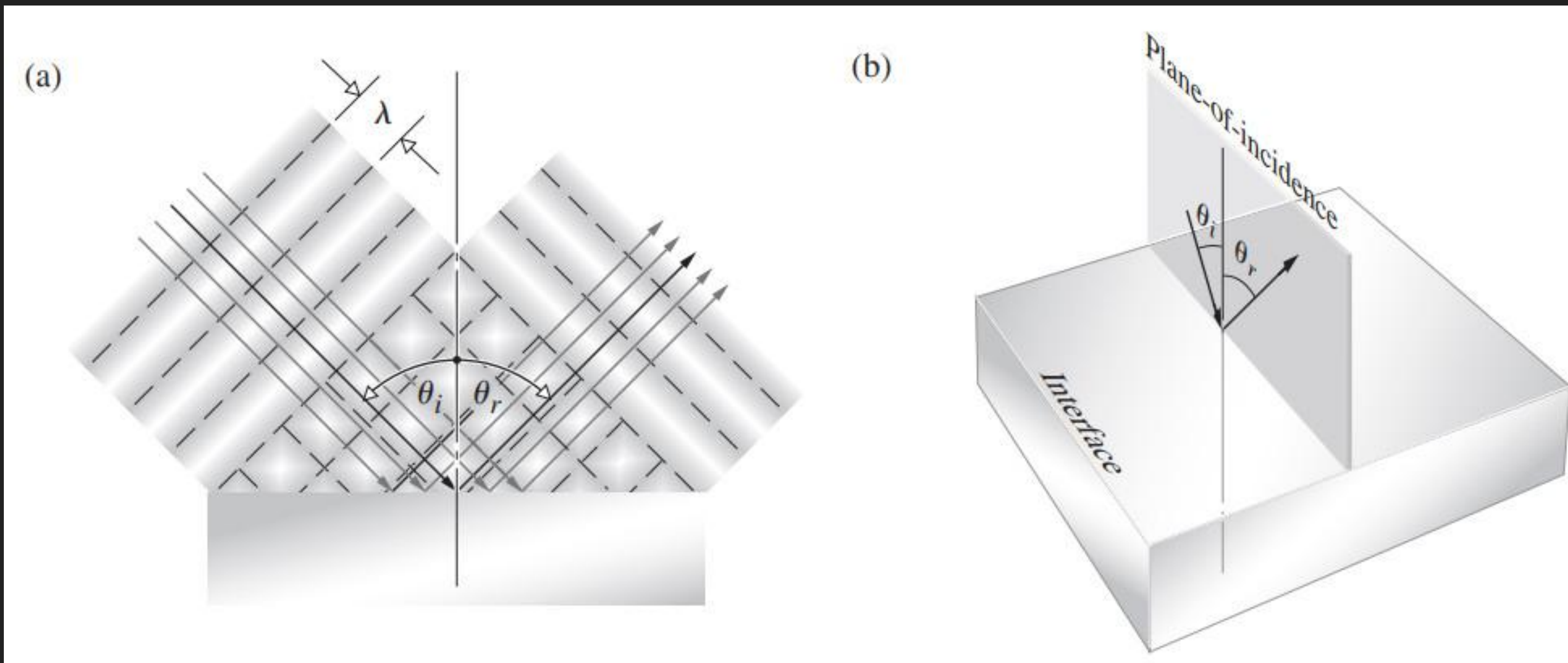
- Maxwell's equations capture the physics of the EM fields, whose disturbances are transverse waves that travel with the constant speed of light in vacuum.
- EM waves travel with a slower speed in matter, which is captured by a frequency-dependent, complex index of refraction. Microscopically this can be explained via the Lorentz model (many driven oscillators).
- Refraction can be understood on a microscopic level as the repeated elastic scattering of EM waves, which interfere to alter the apparent phase velocity.

Reflection I

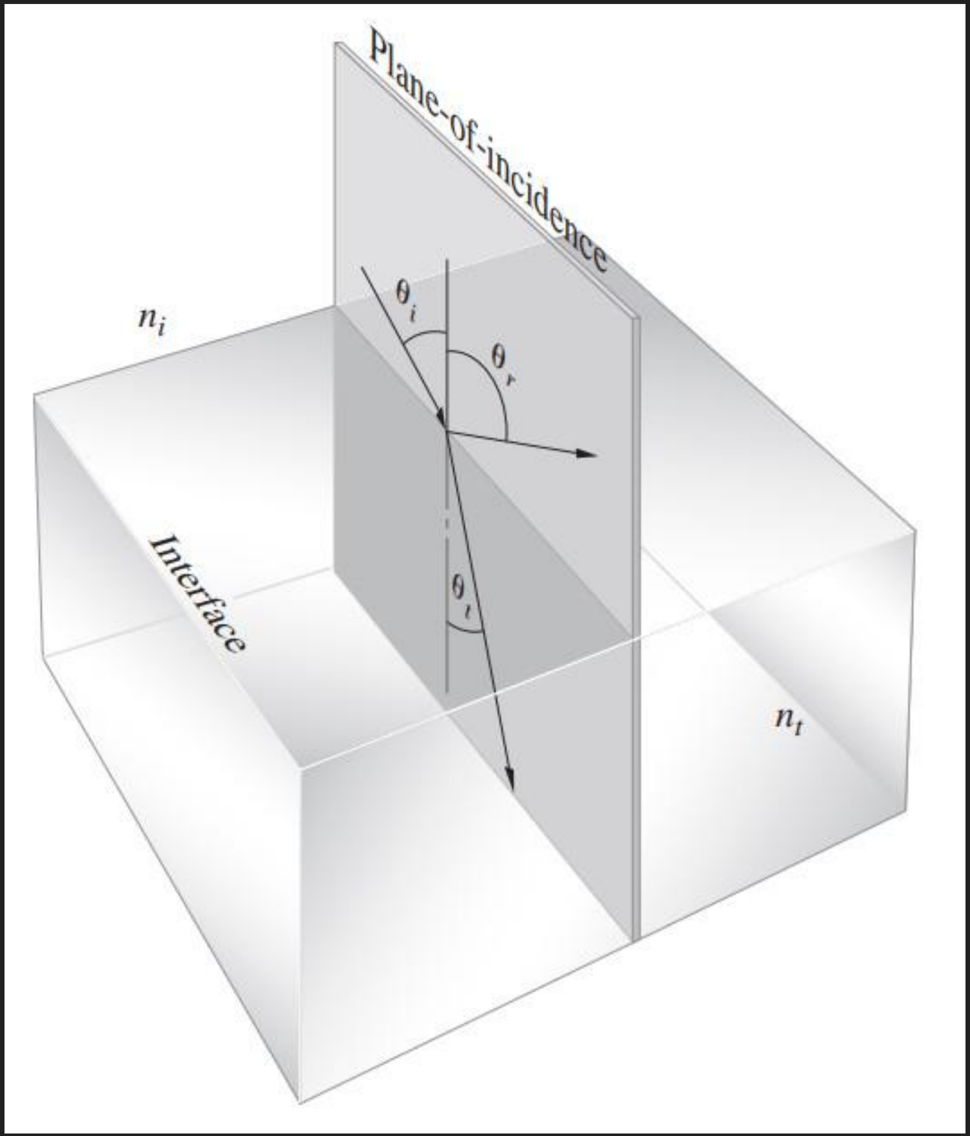




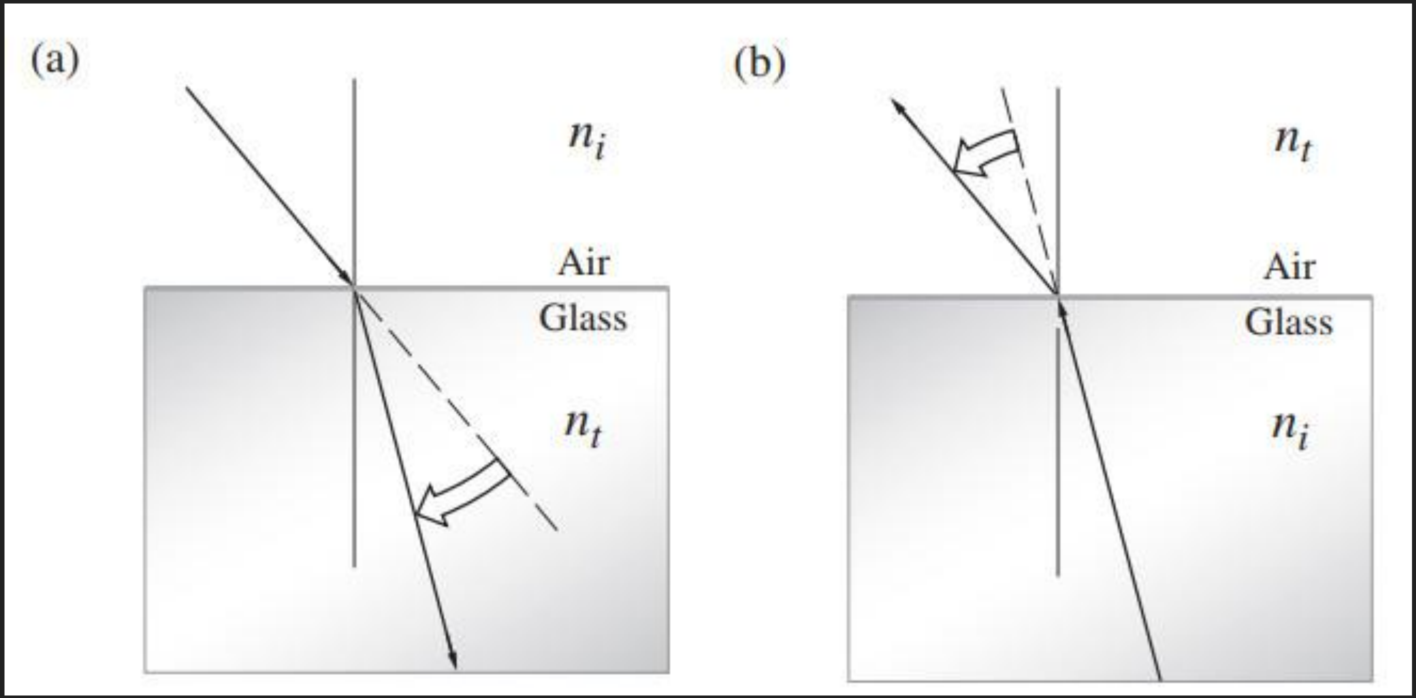
Reflection II



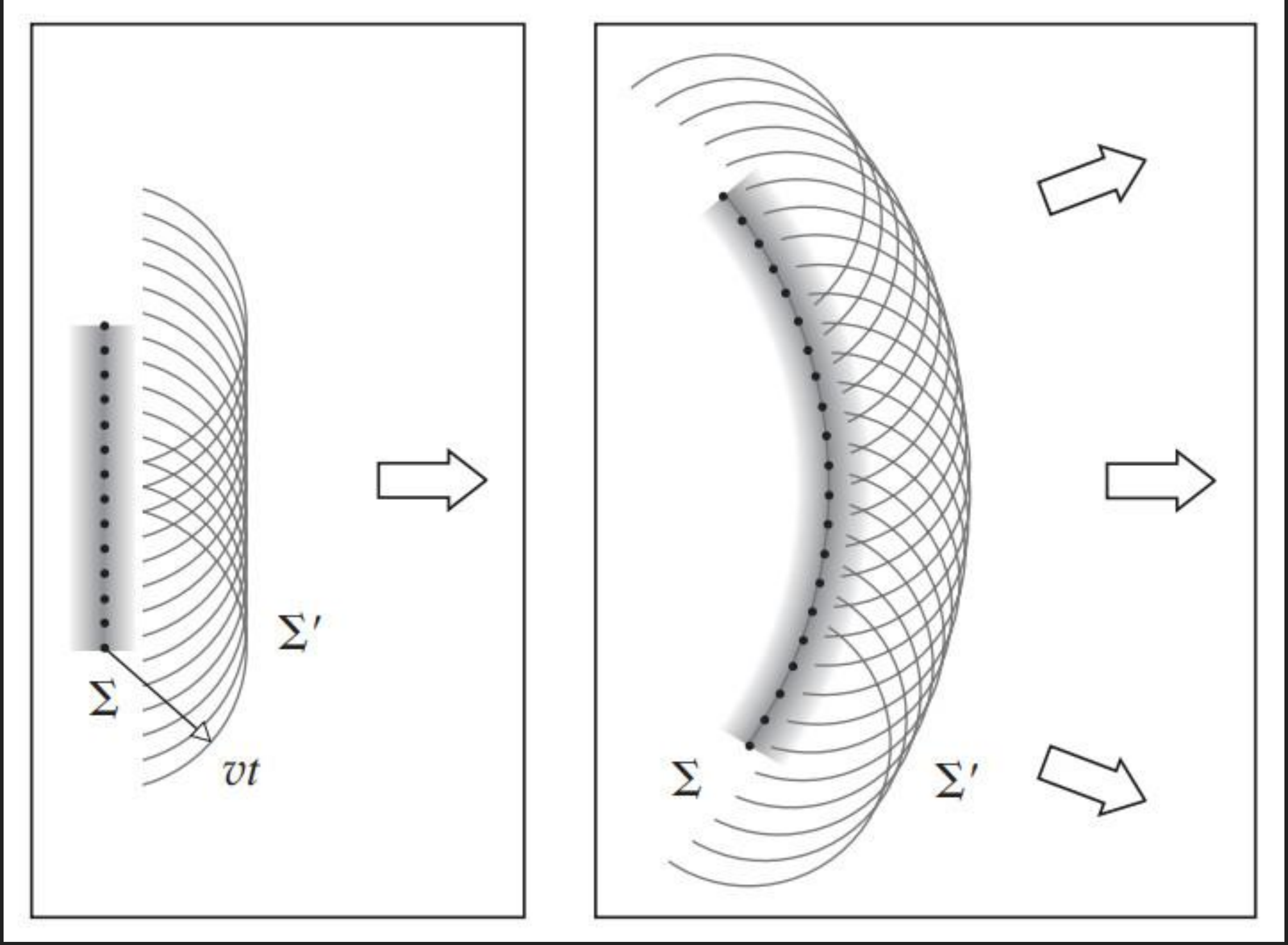
Refraction I



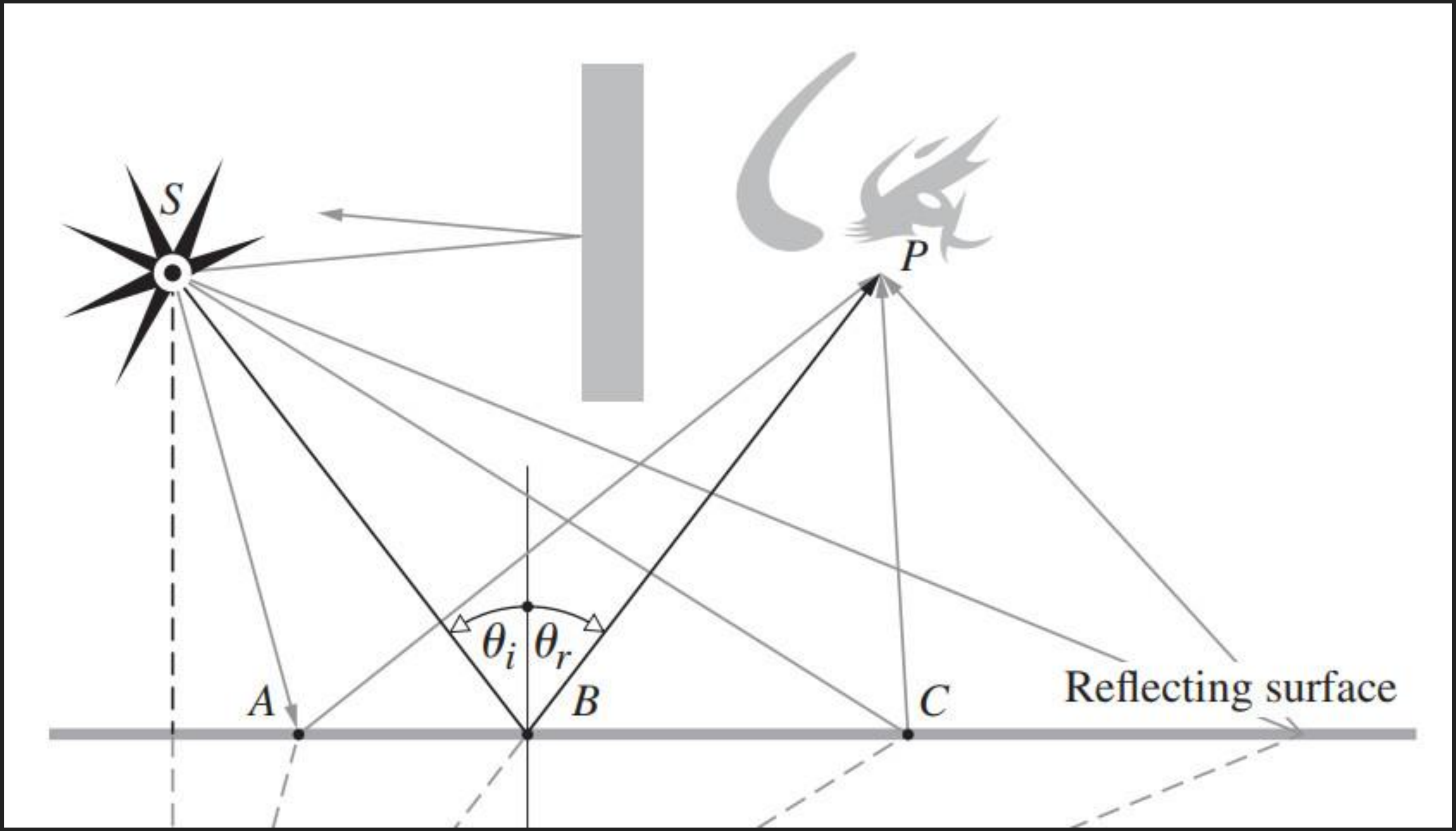
Refraction II



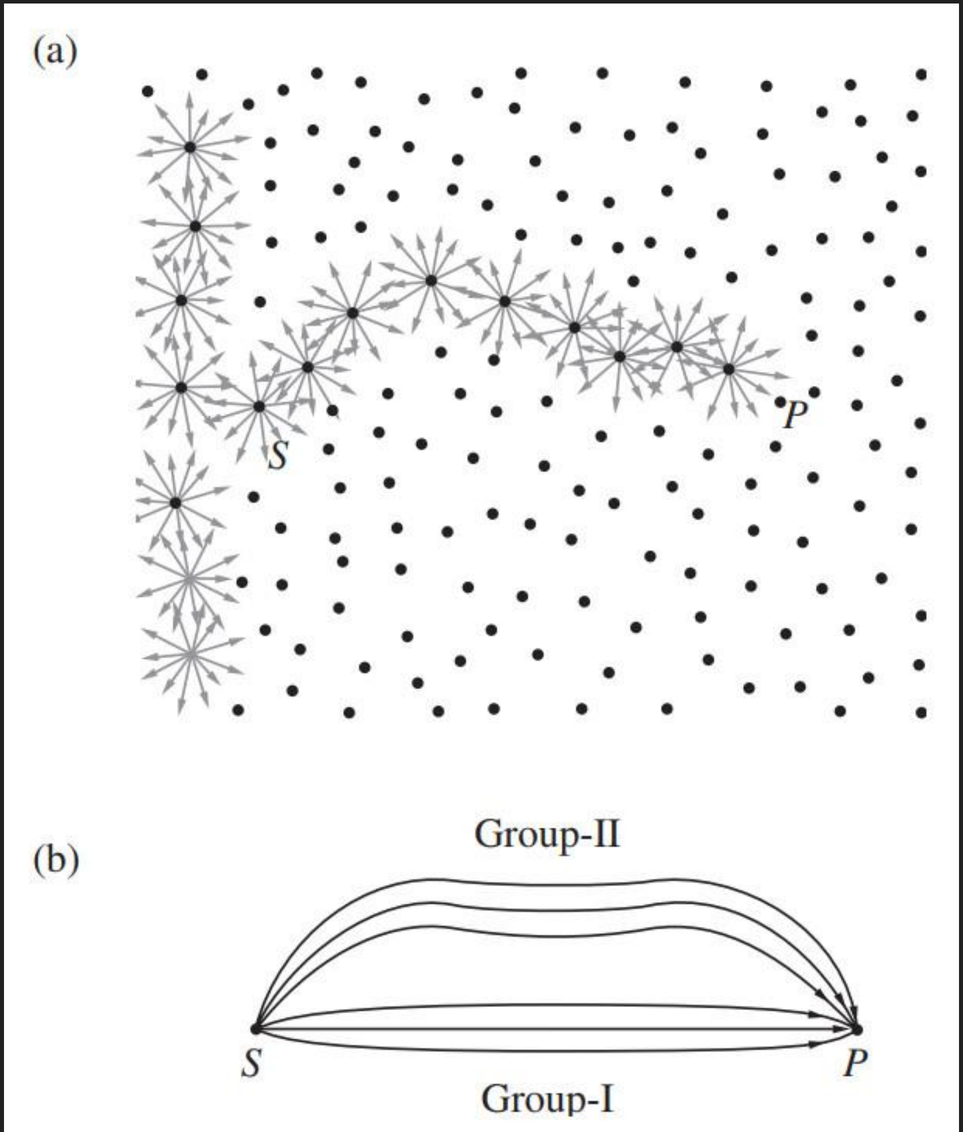
Huygens' principle



Fermat's principle



Stationary paths



Summary Lecture 3

- Based on the concepts of scattering and interference, it is possible to derive the reflection/refraction law by following the motion of wave fronts:

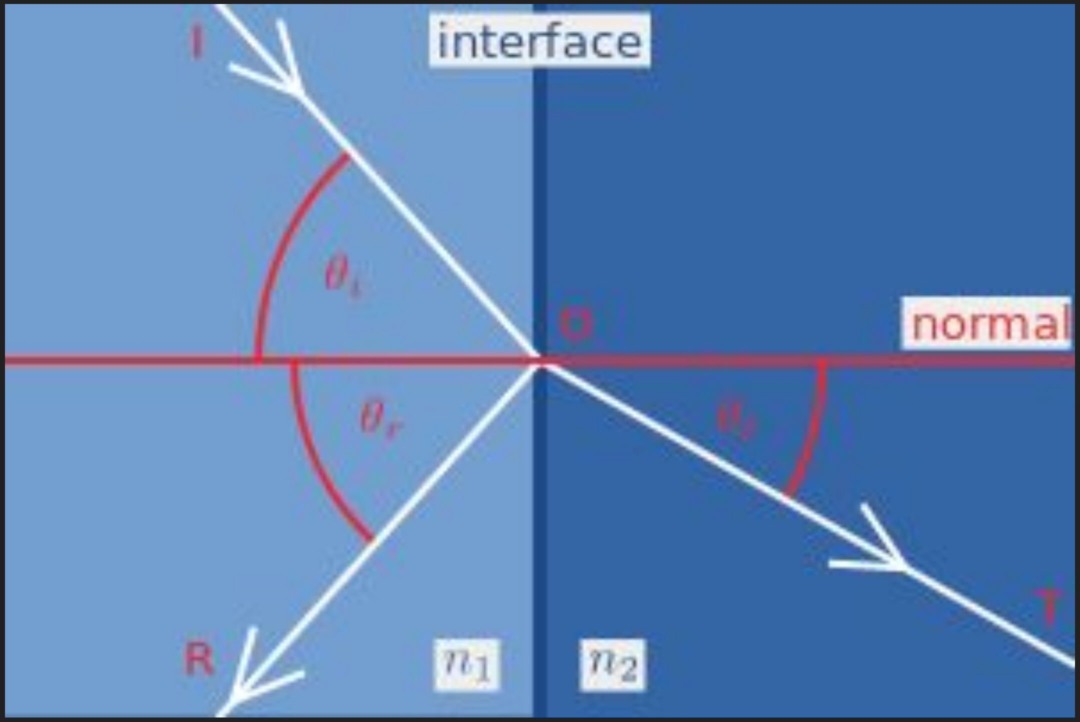
$$\theta_i = \theta_r \qquad\qquad n_i \sin\theta_i = n_t \sin\theta_t$$

- Huygens' principle states that each point on a wave front is the source of an outgoing spherical wave. Their interference describes the wave propagation.
- Fermat's principle gives a new perspective on light propagation: light chooses the stationary path.

PHYS 434 Optics

Lecture 4: EM approach, Fresnel Equations, Total Internal Reflection

Reading: 4.6, 4.7



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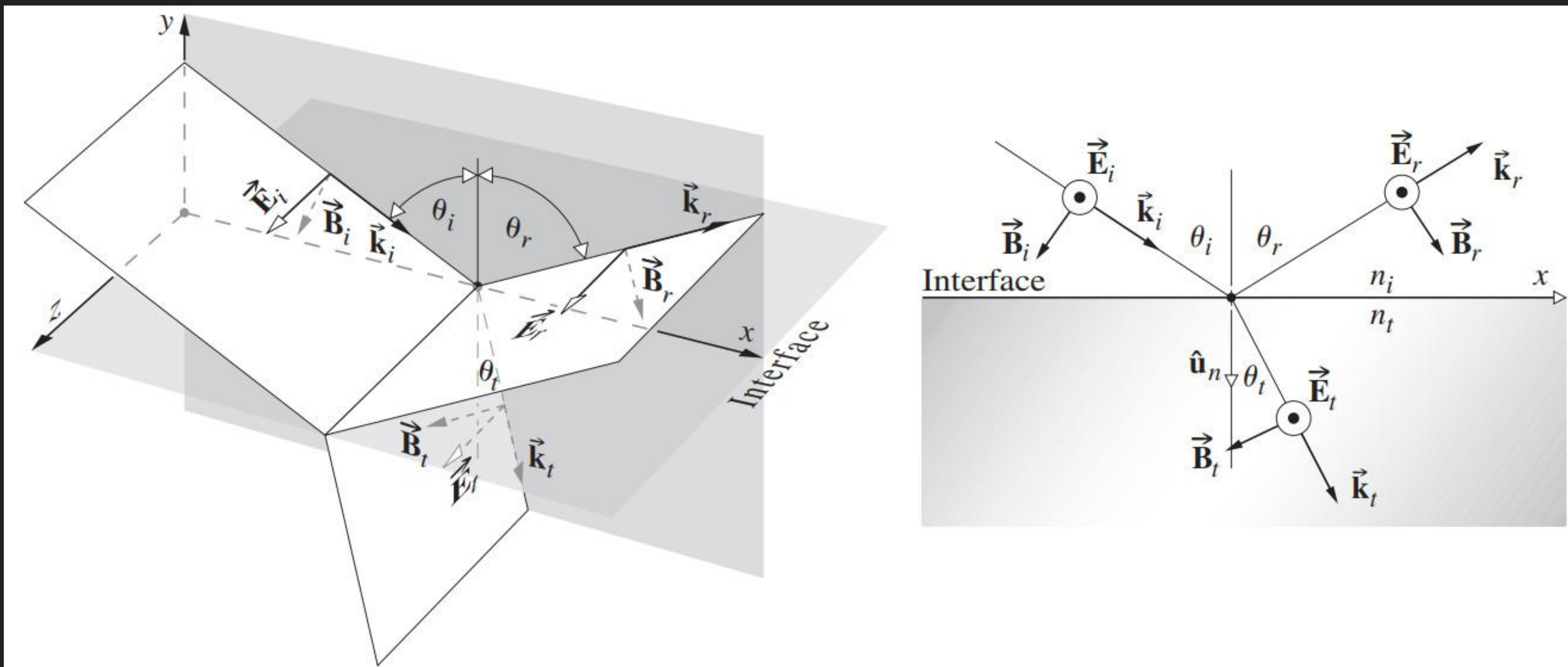
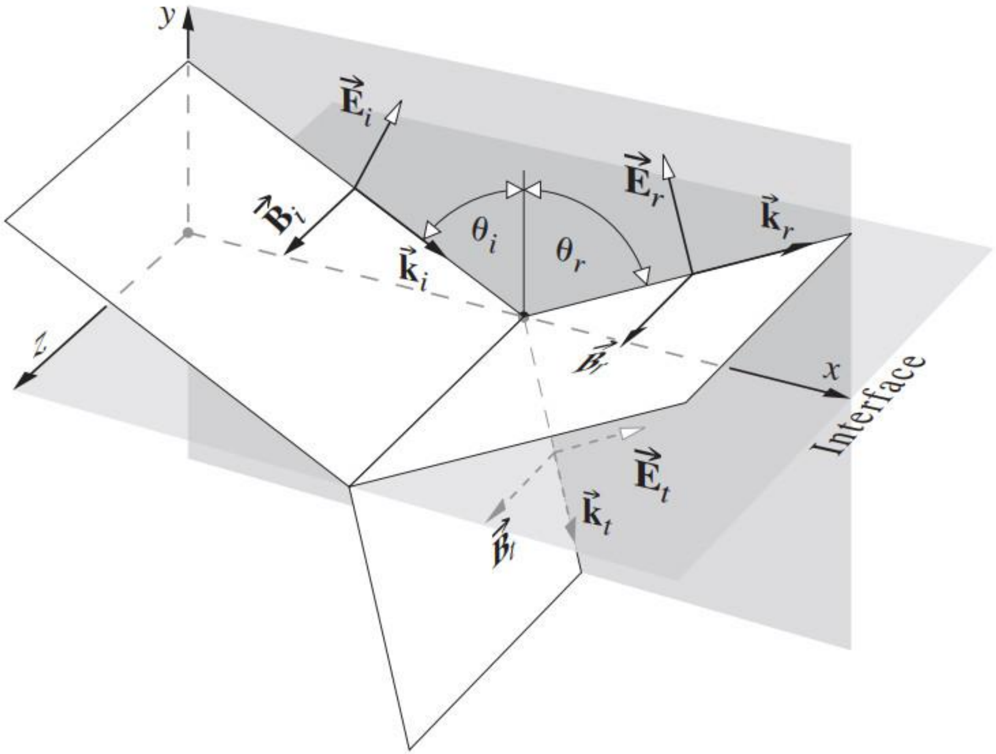
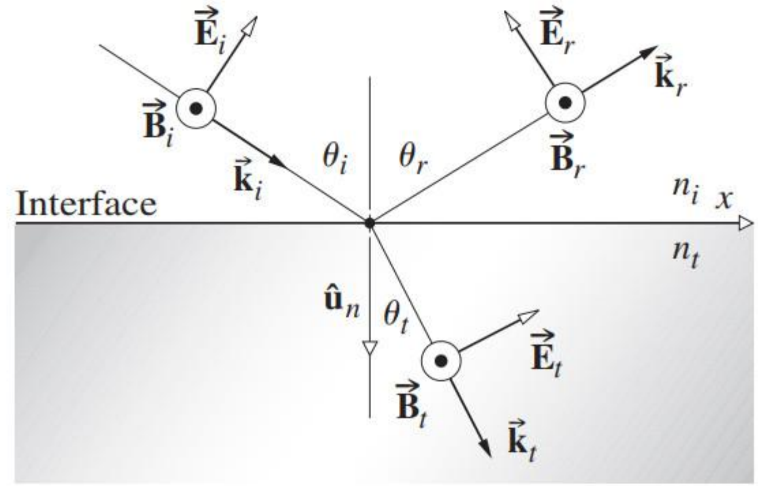
Fresnel equations I

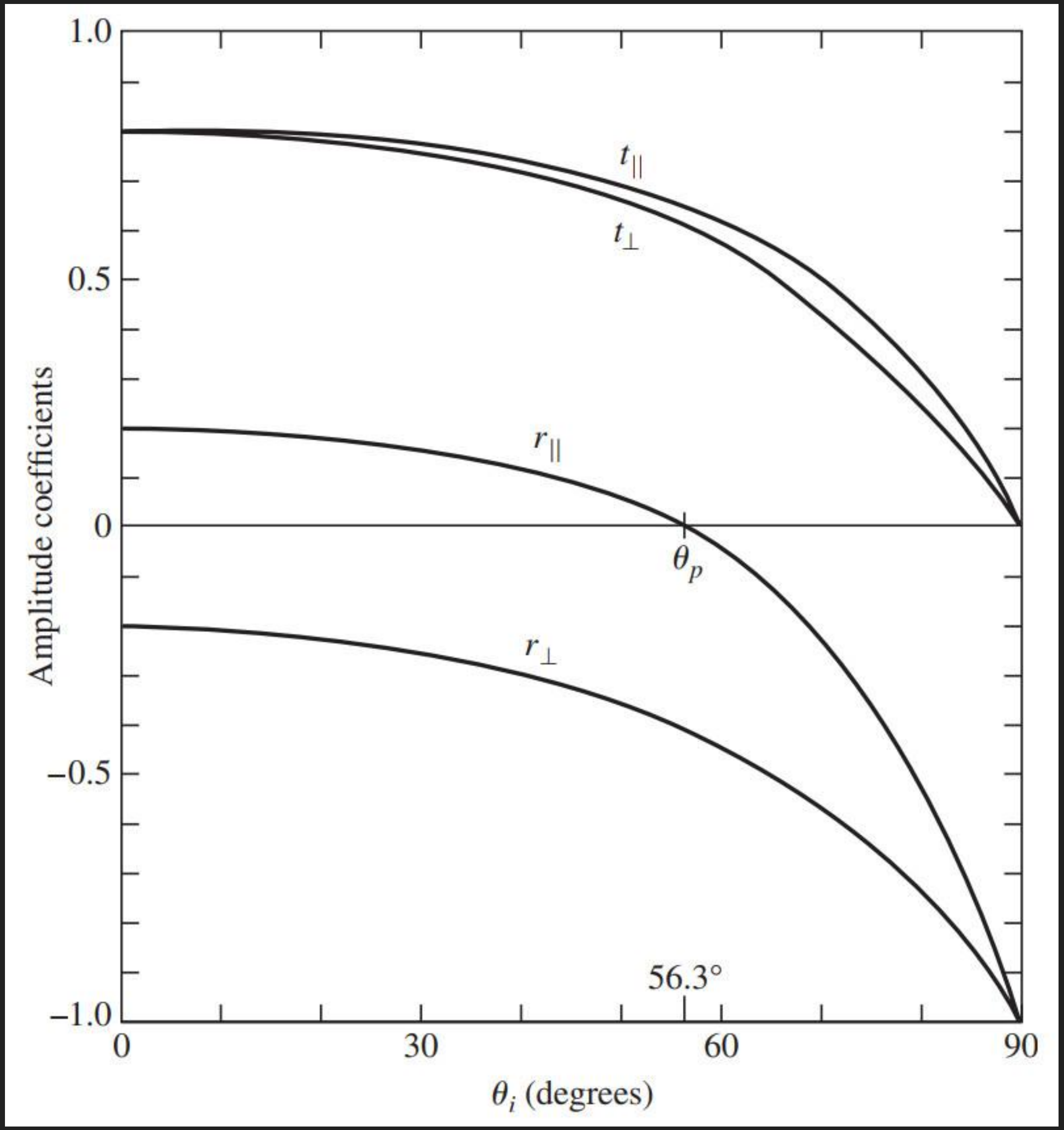


Fresnel equations II



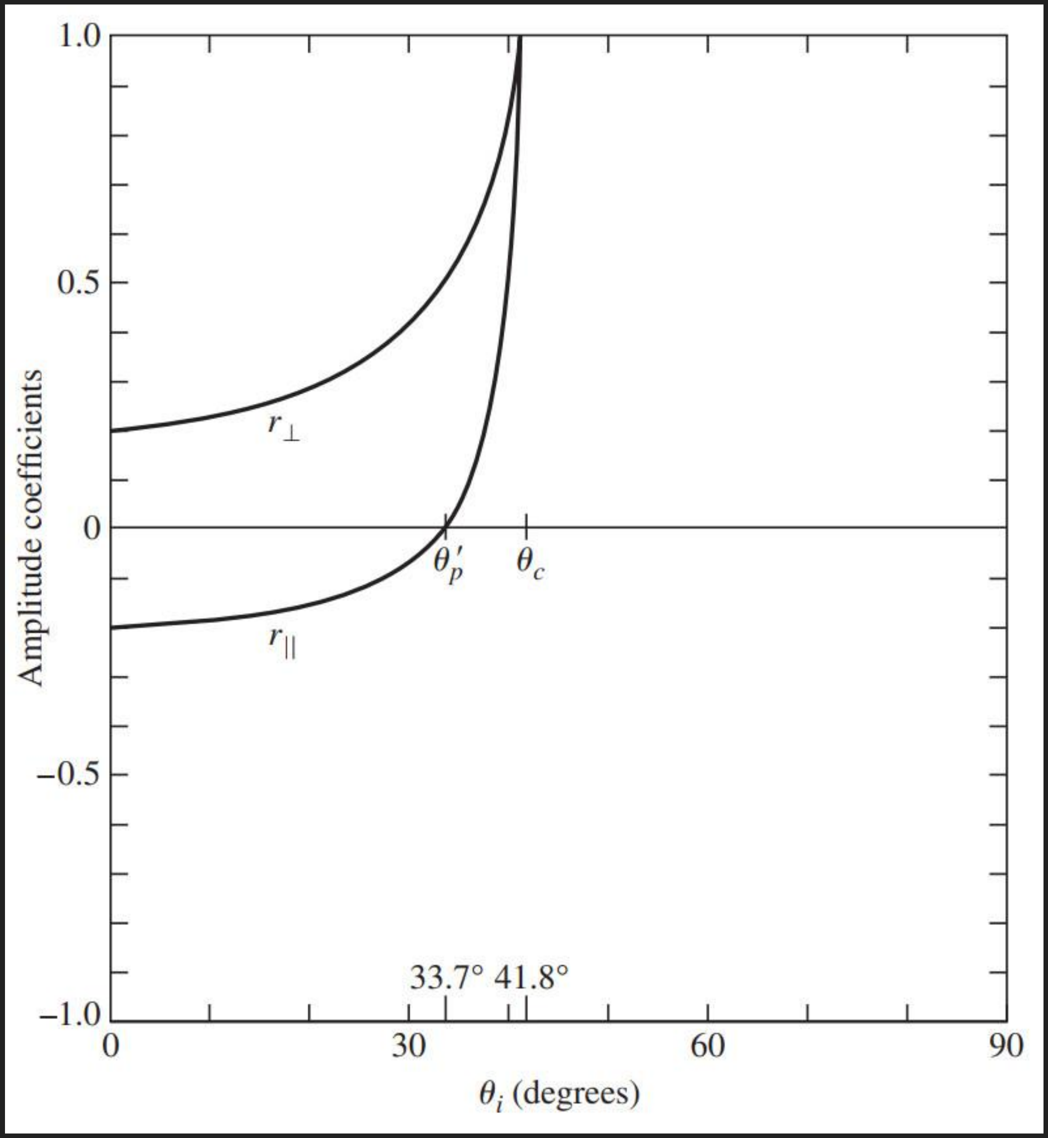


Amplitude coefficients I



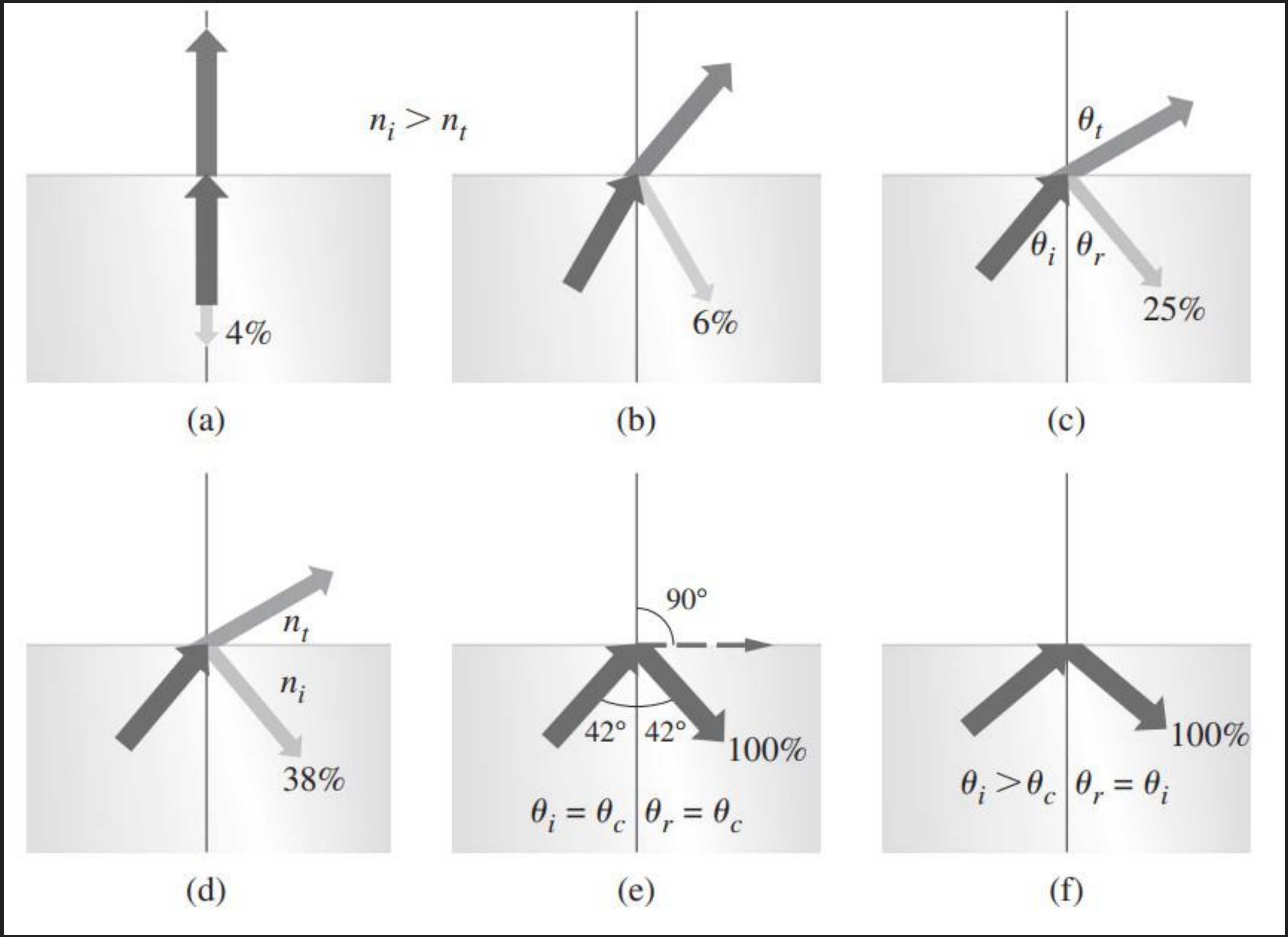
- External reflection for air-glass interface

Amplitude coefficients II



- Internal reflection for glass-air interface

Total internal reflection



Summary Lecture 4

- Based on continuity conditions for the electric and magnetic field components, as well as the laws of reflection and refraction, we can derive ratios of the incident and reflected/transmitted wave amplitude.
- The resulting Fresnel equations provide the means to quantitatively study how an incident EM wave is affected by an interface and 'proof' several of the concepts, we have discussed so far.
- We calculated the power transferred in this process by addressing the reflectance and transmittance.