Magnetic field evolution in superconducting neutron stars

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Magnetic Fields in Neutron Stars

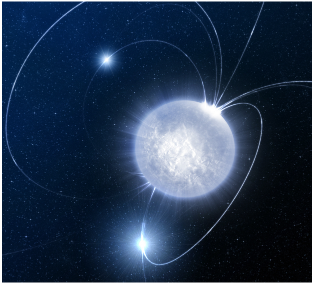

Figure 1: Artistic impression of a neutron star and its magnetic dipole field.

- Inferred magnetic dipole field strengths reach up to $10^{15}\,\mathrm{G}$ for **magnetars**. Such fields strongly influence the dynamics.
- Long-term **field evolution** could explain
  - observed field changes in pulsars (Narayan & Ostriker, 1990)
  - high activity of magnetars (Thompson & Duncan, 1995)
  - neutron star 'metamorphosis' (Viganò et al., 2013)
- Mechanisms causing magnetic field evolution are poorly understood. (Goldreich & Reisenegger, 1992; Glampedakis, Jones & Samuelsson, 2011, e.g.)

What happens if we take **core** superconductivity into account?

Standard Resistive MHD

- In a proton-electron plasma, the **frictional coupling** force is given by

$$F_{\mathrm{e}}^i = \frac{n_{\mathrm{e}} m_{\mathrm{e}}}{\tau_{\mathrm{e}}} \left(v_{\mathrm{e}}^i - v_{\mathrm{p}}^i \right) \equiv -\frac{m_{\mathrm{e}}}{e\tau_{\mathrm{e}}} J^i, \tag{1}$$

with the coupling timescale τ_{e} and macroscopic current J^i.

- The electron Euler equation provides a generalised Ohm's law for the macroscopic electric field. Together with Faraday's and Ampère's laws this leads to the **resistive MHD induction equation**,

$$\partial_t B^i = \epsilon^{ijk} \nabla_j \epsilon_{klm} \left[v_{\mathrm{p}}^l B^m - \frac{c^2}{4\pi\sigma_{\mathrm{e}}} \nabla^l B^m - \frac{m_{\mathrm{p}} c}{4\pi e \rho_{\mathrm{p}}} \epsilon^{lst} (\nabla_s B_t) B^m \right], \tag{2}$$

$\Rightarrow$ Flux freezing, Ohmic decay and conservative Hall evolution with

$$\tau_{\mathrm{Ohm}} = \frac{4\pi\sigma_{\mathrm{e}} L^2}{c^2} \approx 2.4 \times 10^{13}\,\mathrm{yr}, \qquad \tau_{\mathrm{Hall}} = \frac{4\pi e \rho_{\mathrm{p}} L^2}{m_{\mathrm{p}} c B} \approx 1.9 \times 10^{10}\,\mathrm{yr}. \tag{3}$$

Quantum Condensates – Type-II Superconductivity

- Equilibrium stars with $10^6 - 10^8\,\mathrm{K}$ are cold enough to contain **superfluid** neutrons and **superconducting** protons. The macroscopic quantum states influence the stars' dynamics.

- Type of superconductivity depends on the characteristic lengthscales. Estimates predict a **type-II state** (Baym, Pethick & Pines, 1969; Mendell, 1991, e.g.)

$$\kappa = \frac{\lambda}{\xi} \approx 2\,\rho_{14}^{-5/6} \left(\frac{x_{\mathrm{p}}}{0.05}\right)^{-5/6} \left(\frac{T_{\mathrm{cp}}}{10^9\,\mathrm{K}}\right) > \frac{1}{\sqrt{2}}. \qquad (4)$$

- Flux is allowed to enter fluid in form of **quantised fluxtubes**. They are arranged in a hexagonal array. Each fluxline carries a unit of flux,

$$\phi_0 = \frac{hc}{2e} \approx 2 \times 10^{-7}\,\mathrm{G\,cm}^2. \qquad (5)$$

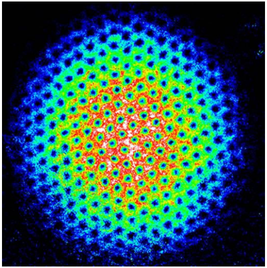

Figure 2: Vortex array in a rotating, dilute BEC of Rubidium atoms (Engels et al., 2002).

Superconducting MHD

- The macroscopic **Euler equations** for the superfluid neutrons and the combined proton-electron fluid are (Glampedakis, Andersson & Samuelsson, 2011)

$$\left(\partial_t + v_{\rm n}^j \nabla_j\right) \left[v_{\rm n}^i + \varepsilon_{\rm n} w_{\rm np}^i\right] + \nabla^i \tilde{\Phi}_{\rm n} + \varepsilon_{\rm n} w_{\rm pn}^j \nabla^i v_j^{\rm n} = f_{\rm mf}^i + f_{\rm mag,n}^i, \qquad (6)$$

$$\left(\partial_t + v_{\rm p}^j \nabla_j\right) \left[v_{\rm p}^i + \varepsilon_{\rm p} w_{\rm pn}^i\right] + \nabla^i \tilde{\Phi}_{\rm p} + \varepsilon_{\rm p} w_{\rm np}^j \nabla^i v_j^{\rm p} = -\frac{n_{\rm n}}{n_{\rm p}} f_{\rm mf}^i + f_{\rm mag,p}^i, \qquad (7)$$

with $w_{\rm xy}^i \equiv v_{\rm x}^i - v_{\rm y}^i$. The equations are modified by **additional force terms**, $f_{\rm mf}^i$ and $f_{\rm mag,x}^i$, due to vortices/fluxtubes and **entrainment**, $\varepsilon_{\rm x}$.

- They are supplemented by continuity equations and the Poisson equation

$$\partial_t n_{\rm x} + \nabla_i \left(n_{\rm x} v_{\rm x}^i\right) = 0, \qquad \nabla^2 \Phi = 4\pi G \rho. \qquad (8)$$

Evolution equation for the magnetic field of a type-II superconductor?

Conventional Mutual Friction

- Standard **'resistivity'** due to electrons scattering off magnetic fields of individual fluxtubes (Alpar, Langer & Sauls, 1984) results in **macroscopic force**

$$F_{\mathrm{e}}^i = \mathcal{N}_{\mathrm{p}} f_{\mathrm{d}}^i = \mathcal{N}_{\mathrm{p}} \rho_{\mathrm{p}} \kappa \mathcal{R} \left(v_{\mathrm{e}}^i - u_{\mathrm{p}}^i \right), \tag{9}$$

with fluxtube density $\mathcal{N}_{\mathrm{p}}$, electron drag f_{d}^i and fluxtube velocity u_{p}^i.

Figure 3: Fluxtubes can be envisaged as tiny, rotating tornadoes. Different forces determine their motion (NOAA Photo Library).

- The dimensionless drag coefficient is (Mendell, 1991)

$$\mathcal{R} \sim 2.3 \times 10^{-4} \rho_{14}^{1/6} \left(\frac{x_{\mathrm{p}}}{0.05} \right)^{1/6} \ll 1. \tag{10}$$

- Rewrite u_{p}^i in terms of fluid variables to obtain a macroscopic equation. We use a mesoscopic **force balance** for an individual fluxtube (Hall & Vinen, 1956).

Results I - Superconducting Induction Equation

- Eliminating u_{p}^i leads to the force

$$F_{\mathrm{e}}^i \approx -\frac{H_{\mathrm{c}1} B}{4\pi} \frac{\mathcal{R}}{1+\mathcal{R}^2} \left(\mathcal{R} \hat{B}^j \nabla_j \hat{B}^i + \epsilon^{ijk} \hat{B}_j \hat{B}^l \nabla_l \hat{B}_k \right), \tag{11}$$

with the lower critical field of superconductivity $H_{\mathrm{c}1}$ and $B^i = B\hat{B}^i$.

- As before, combine (11) with Euler equation and Faraday's law to obtain a **superconducting induction equation** for standard mutual friction,

$$\boxed{\partial_t B^i \approx \epsilon^{ijk} \nabla_j \left[\epsilon_{klm} \left(v_{\mathrm{p}}^l B^m \right) - \frac{\kappa B}{2\pi} \frac{m_{\mathrm{p}}}{m_{\mathrm{p}}^*} \frac{\mathcal{R}}{1+\mathcal{R}^2} \left(\mathcal{R} \hat{B}^l \nabla_l \hat{B}_k + \epsilon_{klm} \hat{B}^l \hat{B}^s \nabla_s \hat{B}^m \right) \right].}$$

- For $\mathcal{R} \ll 1$, the inertial term dominates and the field is **frozen** to the protons; on large scales electrons, protons and fluxtubes are comoving.

Results II - Conservative/Dissipative Contributions

- Nature of terms in both induction equations is determined by looking at the **evolution** of the **magnetic energy**. For standard MHD, we obtain

$$\frac{\partial \mathcal{E}_{\mathrm{mag}}}{\partial t} = \frac{B_i}{4\pi}\frac{\partial B^i}{\partial t} = \frac{1}{c}\, J^i \epsilon_{ijk}\, v_{\mathrm{p}}^j B^k - \frac{J^2}{\sigma_{\mathrm{e}}} - \nabla^i \Sigma_i. \tag{12}$$

  The Hall term vanishes, while Ohmic diffusion causes energy loss $\propto J^2$.

- In the **superconducting case**, we have $\mathcal{E}_{\mathrm{mag,sc}} = H_{\mathrm{c1}} B/2\pi$ and

$$\frac{\partial \mathcal{E}_{\mathrm{mag,sc}}}{\partial t} = \frac{B}{2\pi}\frac{\partial H_{\mathrm{c1}}}{\partial t} + \frac{H_{\mathrm{c1}}}{2\pi}\left(\mathcal{J}_\perp^i \epsilon_{ijk} v_{\mathrm{p}}^j B^k - \frac{\kappa B}{2\pi}\frac{m_{\mathrm{p}}}{m_{\mathrm{p}}^*}\frac{\mathcal{R}}{1+\mathcal{R}^2}\mathcal{J}_\perp^2 - \nabla^i \Sigma_i\right), \tag{13}$$

  with $\mathcal{J}^i \equiv \epsilon^{ijk}\nabla_j \hat{B}_k$ decomposed into $\mathcal{J}^i \equiv \mathcal{J}_\parallel \hat{B}^i + \mathcal{J}_\perp^i$. The first term shows that changing the superconducting properties alters the magnetic energy.

Results III - Timescales

$$\boxed{\partial_t B^i \approx \epsilon^{ijk}\nabla_j\left[\epsilon_{klm}\left(v_{\rm p}^l B^m\right) - \frac{\kappa B}{2\pi}\frac{m_{\rm p}}{m_{\rm p}^*}\frac{\mathcal{R}}{1+\mathcal{R}^2}\left(\mathcal{R}\hat{B}^l\nabla_l\hat{B}_k + \epsilon_{klm}\hat{B}^l\hat{B}^s\nabla_s\hat{B}^m\right)\right].}$$

- Similar to the Hall evolution of resistive MHD, the second term is **conservative**. The last term is **dissipative** (like Ohmic decay) and decreases the magnetic energy of the superconducting mixture $\propto \mathcal{J}_\perp^2$.

- Extract the **dominant timescales** from the induction equation

$$\tau_{\rm diss} = \frac{2\pi L^2}{\kappa}\frac{1+\mathcal{R}^2}{\mathcal{R}}\frac{m_{\rm p}^*}{m_{\rm p}} \approx 3.1\times 10^{11}\,{\rm yr}, \qquad \tau_{\rm cons} = \frac{\tau_1}{\mathcal{R}} \approx 1.3\times 10^{15}\,{\rm yr}. \tag{14}$$

Comparison gives:

$$\frac{\tau_{\rm diss}}{\tau_{\rm Ohm}} \approx 1.3\times 10^{-2}, \qquad \frac{\tau_{\rm cons}}{\tau_{\rm Hall}} \approx 6.8\times 10^{4}. \tag{15}$$

Conclusions and Open Questions

- Analogous to standard MHD, we chose one mesoscopic effect to derive a **macroscopic induction equation** for the superconducting mixture $\Rightarrow$ flux freezing, dissipative/conservative contributions are present.
- $\tau_{\rm diss}$ and $\tau_{\rm cons}$ are **notably longer** than the typical spin-down ages $\Rightarrow$ conventional mutual friction cannot explain observed field changes due to minimum dissipation timescale $\tau_{\mathsf{min}} \approx 1.4 \times 10^8\,\mathrm{yr}$ for $\mathcal{R} = 1$.
- For shorter timescales, different dissipative mechanisms are necessary $\Rightarrow$ typical candidate for strong coupling is **vortex-fluxtube 'pinning'**
- **Key issue**: We discuss bulk fluid evolution but neglect surface terms $\Rightarrow$ effects due to the **crust-core interface** are not included. Physics are poorly understood but could be very important for neutron stars.

Magnetic Energy

- In standard MHD, the **Lorentz force** contains tension and pressure term

$$F^i_{\mathrm{L}} = \frac{1}{4\pi}\left[B_j \nabla^j B^i - \frac{1}{2}\nabla^i \left(B_k B^k\right)\right]. \tag{16}$$

The work is then given by

$$W_{\mathrm{L}} = \int r_i F^i_{\mathrm{L}} \,\mathrm{d}V = \int \frac{B^2}{8\pi}\,\mathrm{d}V \equiv \int \mathcal{E}_{\mathrm{mag}}\,\mathrm{d}V. \tag{17}$$

- For a **superconductor**, the total magnetic force has to be changed to (Easson & Pethick, 1977; Glampedakis, Andersson & Samuelsson, 2011)

$$F^i_{\mathrm{mag}} = \frac{1}{4\pi}\left[B_j \nabla^j H^i_{\mathrm{c1}} - \nabla^i \left(\rho_{\mathrm{p}} B \frac{\partial H_{\mathrm{c1}}}{\partial \rho_{\mathrm{p}}}\right)\right], \tag{18}$$

where $H^i_{\mathrm{c1}} = H_{\mathrm{c1}} \hat{B}^i$. Integration gives for the energy of the bulk fluid

$$W_{\mathrm{mag}} = \int r_i F^i_{\mathrm{mag}} \,\mathrm{d}V = \int \frac{H_{\mathrm{c1}} B}{2\pi}\,\mathrm{d}V \equiv \int \mathcal{E}_{\mathrm{mag,sc}}\,\mathrm{d}V. \tag{19}$$

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