Fluxtube dynamics in neutron star cores

- Implications for magnetic field evolution

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Background

Magnetic fields

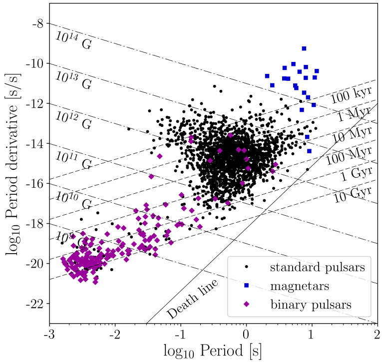


Figure 1: Neutron star $P\dot{P}$ diagram.

- Inferred magnetic dipole field strengths reach up to 10^{15} G for magnetars. Such fields strongly influence the stars' dynamics.
- Long-term **field evolution** could explain
  - observed field changes in pulsars
  - high activity of magnetars
  - neutron star 'metamorphosis'
- Mechanisms causing magnetic field evolution are poorly understood.

What happens if we account for core superconductivity?

Macroscopic quantum states

- Equilibrium stars have $10^6 - 10^8\,\mathrm{K}$, while for nucleons $T_\mathrm{F} \sim 10^{12}\,\mathrm{K} \Rightarrow$ they are cold enough to contain **superfluid** neutrons and **superconducting** protons. Cooper pair formation occurs due to an **attractive contribution** to the nucleon-nucleon interaction.

- The type of superconductivity depends on the characteristic lengthscales. Estimates predict a **type-II state** (Baym, Pethick & Pines, 1969b; Mendell, 1991)

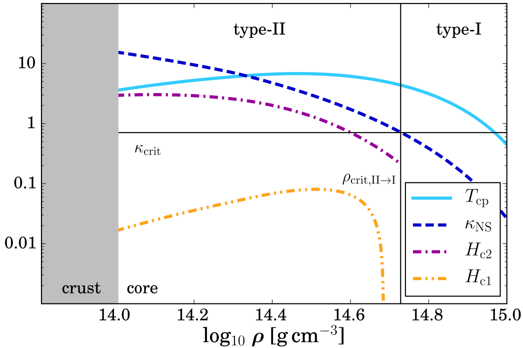
$$\kappa_\mathrm{NS} = \frac{\lambda}{\xi_\mathrm{ft}} \approx 3 \left(\frac{m_\mathrm{p}^*}{m}\right)^{\frac{3}{2}} \rho_{14}^{-\frac{5}{6}} \left(\frac{x_\mathrm{p}}{0.05}\right)^{-\frac{5}{6}} \left(\frac{T_\mathrm{cp}}{10^9\,\mathrm{K}}\right) > \frac{1}{\sqrt{2}}, \tag{1}$$

$$H_\mathrm{c1} = \frac{4\pi\mathcal{E}_\mathrm{ft}}{\phi_0} \approx 1.9 \times 10^{14} \left(\frac{m}{m_\mathrm{p}^*}\right) \rho_{14} \left(\frac{x_\mathrm{p}}{0.05}\right) \mathrm{G}, \tag{2}$$

$$H_\mathrm{c2} = \frac{\phi_0}{2\pi\xi_\mathrm{ft}^2} \approx 2.1 \times 10^{15} \left(\frac{m_\mathrm{p}^*}{m}\right)^2 \rho_{14}^{-\frac{2}{3}} \left(\frac{x_\mathrm{p}}{0.05}\right)^{-\frac{2}{3}} \left(\frac{T_\mathrm{cp}}{10^9\,\mathrm{K}}\right)^2 \mathrm{G}. \tag{3}$$

Type-II superconductivity

Figure 2: Density-dependent parameters of NS superconductivity calculated for the NRAPR effective equation of state (Steiner et al., 2005).



Shown are κ_{NS}, T_{cp} (normalised to $10^9\,\mathrm{K}$), H_{c2} and H_{c1} (normalised to $10^{16}\,\mathrm{G}$). The horizontal and vertical line mark $\kappa_{\mathrm{crit}} = 1/\sqrt{2}$ and $\rho_{\mathrm{crit,II}\to\mathrm{I}}$, respectively.

Background

Fluxtubes

- Magnetic flux enters the system in the form of **quantised fluxtubes**, arranged in a hexagonal array. Each fluxtube carries a unit of flux,

$$\phi_0 = \frac{hc}{2e} \approx 2 \times 10^{-7}\,\mathrm{G\,cm}^2. \tag{4}$$

- All flux quanta add up to the macroscopic magnetic induction B in the star's core.

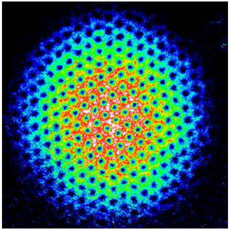


Figure 3: Vortex array in a rotating BEC (Engels et al., 2002).

- Relate B to the fluxtube surface density and interfluxtube distance:

$$\mathcal{N}_{\rm ft} = \frac{B}{\phi_0} \approx 4.8 \times 10^{18}\, B_{12}\,\mathrm{cm}^{-2}, \qquad d_{\rm ft} \simeq \mathcal{N}_{\rm ft}^{-\frac{1}{2}} \approx 4.6 \times 10^{-10}\, B_{12}^{-\frac{1}{2}}\,\mathrm{cm}. \tag{5}$$

Magnetic field evolution is linked to the motion of fluxtubes!

FT mechanisms — Ohmic dissipation

- Matter **inside fluxtubes** is normal conducting $\Rightarrow$ the dominant coupling is scattering of electrons off normal protons (Baym, Pethick & Pines, 1969a). This process is characterised by an **electrical conductivity**

$$\sigma_{\rm e} = \frac{n_{\rm e} e^2 c \tau}{\hbar k_{\rm Fe}} \approx 5.5 \times 10^{28} \, T_8^{-2} \, \rho_{14}^{\frac{3}{2}} \left(\frac{x_{\rm p}}{0.05} \right)^{\frac{3}{2}} \, {\rm s}^{-1}. \tag{6}$$

- Relate this to **standard Ohmic diffusion**

$$\tau_{\rm Ohm} = \frac{4\pi \sigma_{\rm e} L^2}{c^2} \approx 2.5 \times 10^{13} \, T_8^{-2} \, L_6^2 \, \rho_{14}^{\frac{3}{2}} \left(\frac{x_{\rm p}}{0.05} \right)^{\frac{3}{2}} \, {\rm yr}. \tag{7}$$

- Timescales are very long and further lengthened as fluxtubes only occupy a small fraction of the star's volume, estimated as $B/H_{\rm c2} \sim 10^{-3} B_{12}$.

Resistive drag I

- Fluxtubes are magnetised $\Rightarrow$ electrons can scatter off this magnetic field (Alpar, Langer & Sauls, 1984). In analogy with superfluid hydrodynamics this is generally called **mutual friction**: $\mathbf{f}_\mathrm{d} = \rho_\mathrm{p}\kappa\mathcal{R}(\mathbf{v}_\mathrm{e} - \mathbf{v}_\mathrm{ft})$.

- Using the formalism of Sauls, Stein & Serene (1982) it is possible to determine the corresponding, **dimensionless drag coefficient**. For typical neutron star parameters we obtain

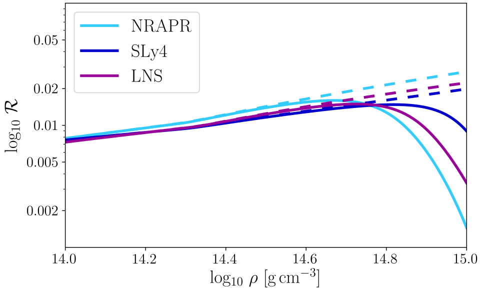
$$\mathcal{R} = \frac{1}{\mathcal{N}_\mathrm{ft}\kappa}\frac{E_\mathrm{Fe}}{mc^2}\frac{1}{\tau} \approx 1.6\times 10^{-2} B_{12}^{-1}\left(\frac{k_\mathrm{Fe}}{0.75\,\mathrm{fm}^{-1}}\right)\left(\frac{10^{-15}\,\mathrm{s}}{\tau}\right) \ll 1. \quad (8)$$

- Since $\mathcal{R} \ll 1$, this is referred to as the limit of weak mutual friction.

For a given EoS and superconducting gap model, $\mathcal{R}$ can be calculated as a function of the star's density.

Resistive drag II

- Resistive drag coefficients for three different EoS (Chamel, 2008) and a standard proton gap parametrisation (Ho, Glampedakis & Andersson, 2012).



- Approximate solution often found in the literature is independent of Δ:

$$\mathcal{R} \approx \frac{3\pi^2}{64} \frac{1}{\lambda k_{\rm Fe}} \approx 7.9 \times 10^{-3} \left(\frac{m}{m_{\rm p}^*} \right)^{1/2} \rho_{14}^{1/6} \left(\frac{x_{\rm p}}{0.05} \right)^{1/6} . \tag{9}$$

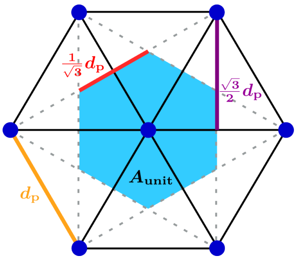
- Deriving a **superconducting induction equation** it can be shown that this mechanism can also not drive fast field evolution (Graber et al., 2015).

Repulsive force I

- Two parallel fluxtubes separated by r_{21} experience a **repulsive force** per unit length

$$\mathbf{F}_{12} = -\nabla\mathcal{E}_{\rm int} = -\frac{\phi_0^2}{8\pi^2\lambda^3}\,K_1\left(\frac{r_{21}}{\lambda}\right)\hat{\boldsymbol{r}}_{21}. \qquad (10)$$

- For a **lattice**, the net force on a single line is obtained by summing individual contributions.



- In the **triangular case** all terms cancel and no field changes takes place.

- However, in a realistic fluxtube lattice the long-range order is likely to be destroyed $\Rightarrow$ a **gradient** in $\mathcal{N}_{\rm ft}$ results in a non-zero net force on the fluxtubes, which would drive field evolution. We expect

$$\mathbf{F}_{\rm rep} = -g(\mathcal{N}_{\rm ft})\nabla\mathcal{N}_{\rm ft}. \qquad (11)$$

FT mechanisms

Repulsive force II

- For standard pulsars with $10^{12} - 10^{13}\,\mathrm{G}$ the problem simplifies because of the hierarchy of the important **lengthscales**, specifically $d_{\rm ft} \simeq r_{21} \gg \lambda$:
  - $K_1\left(r_{21}/\lambda\right)$ can be approximated as a decaying exponential.
  - In the summation only account for the six nearest neighbours.

- In this case, we derive the following **repulsive force**

$$\mathbf{F}_{\rm rep} \simeq -\frac{3\phi_0^2}{32\sqrt{2}\pi^{3/2}}\left(\frac{\sqrt{2}\,\mathcal{N}_{\rm ft}^{-1/2}}{3^{1/4}\lambda}\right)^{\frac{7}{2}} \exp\left[-\frac{\sqrt{2}\,\mathcal{N}_{\rm ft}^{-1/2}}{3^{1/4}\lambda}\right] \nabla\mathcal{N}_{\rm ft}. \tag{12}$$

- Neglecting fluxtube inertia, the **force balance** reads

$$\sum \mathbf{f} = -g(\mathcal{N}_{\rm ft})\nabla\mathcal{N}_{\rm ft} - \rho_{\rm p}\kappa\mathcal{R}\mathbf{v}_{\rm ft} = 0, \tag{13}$$

Repulsive force III

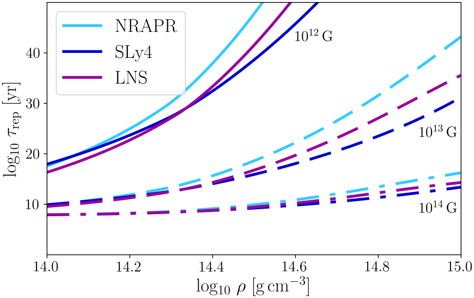
- Combine the force balance with a continuity equation for $\mathcal{N}_{\rm ft}$ to obtain a **non-linear diffusion equation** for the evolution of the fluxtubes/field

$$\partial_t \mathcal{N}_{\rm ft} + \nabla\left(\mathcal{N}_{\rm ft}\,\mathbf{u}_{\rm ft}\right) = \partial_t \mathcal{N}_{\rm ft} - \nabla\left(\frac{\mathcal{N}_{\rm ft}\, g(\mathcal{N}_{\rm ft})}{\rho_{\rm p}\kappa\mathcal{R}}\,\nabla\mathcal{N}_{\rm ft}\right) = 0. \qquad (14)$$

- Extract a timescale for a characteristic lengthscale L:

$$\tau_{\rm rep} = \frac{L^2\rho_{\rm p}\kappa\mathcal{R}}{\mathcal{N}_{\rm ft}\, g(\mathcal{N}_{\rm ft})}. \qquad (15)$$

- Estimates for $L = 10^6\,{\rm cm}$ and different B fields show strong variability with density.



FT mechanisms

Buoyancy I

- Fluxtubes are buoyant as a result of the **magnetic pressure** inside their cores. This creates a radially acting lift force, $f_{\rm b}$, trying to drive fluxtubes out of the core (Muslimov & Tsygan, 1985; Harvey, Ruderman & Shaham, 1986).

- The **buoyancy force** is related to the gradient of the superconducting magnetic pressure. In the limit $B \lesssim H_{\rm c1}$, which approximates the neutron star core, one has $P = H_{\rm c1} B/4\pi$ (Easson & Pethick, 1977). This gives

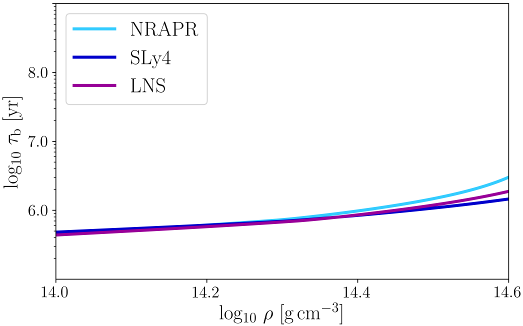
$$f_{\rm b} = \frac{|-\nabla P|}{\mathcal{N}_{\rm ft}} \simeq \frac{H_{\rm c1} B}{\mathcal{N}_{\rm ft} 4\pi L} = \frac{H_{\rm c1}\phi_0}{4\pi L} = \frac{\phi_0^2}{16\pi^2\lambda^2 L} \ln\left(\frac{\lambda}{\xi_{\rm ft}}\right). \tag{16}$$

- Balancing the resistive drag with the buoyancy force, we arrive at a similar non-linear diffusion equation. The respective **timescale** reads

$$\tau_{\rm b} = L^2 \rho_{\rm p} \kappa \mathcal{R} \frac{16\pi^2\lambda^2}{\phi_0^2 \ln(\lambda/\xi_{\rm ft})}. \tag{17}$$

Buoyancy II

- Estimates for $L = 10^6\,\mathrm{cm}$ are of the order of observed field changes.



- Self-consistent **magneto-thermal simulations** of superconducting cores show that buoyancy is too weak to drive field evolution (Elfritz et al., 2016).

Conclusions

- We have studied various mechanisms that are expected to affect the **superconducting fluxtubes** present in the outer neutron star core and calculated characteristic timescales for **realistic equations of state**.
- Resulting field changes act on shortest timescales at low densities, close to the crust-core interface, but are still **too long** to explain observations.
- Many **open problems** remain:
  - How do fluxtubes interact with neutron vortices?
  - Is the outer core in a type-II state after all (maybe type-I)?
  - What is the field configuration right before the phase transition?
- One **possible approach**: Use the **analogy with laboratory systems** to make progress (Graber, Andersson & Hogg, 2017).

Conclusions

Thank you!

Appendix

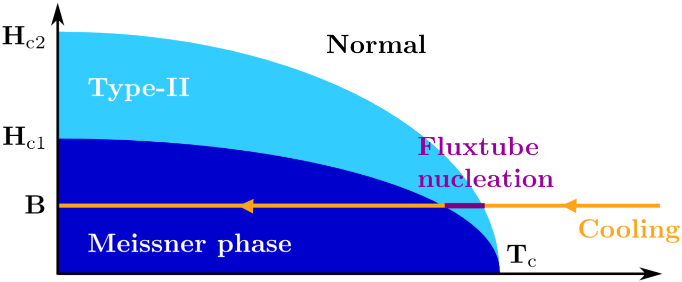


Figure 4: $H(T)$-diagram of a type-II superconductor illustrating the phase transition. As the medium permeated by the induction $B < H_{c1}$ is cooled down, it follows the yellow line from right to left. Below the transition temperature, T_c, magnetic flux (continuously distributed in the normal state) is first nucleated into fluxtubes. These are subsequently expelled if the matter is cooled further and a flux-free Meissner state is formed.

Appendix

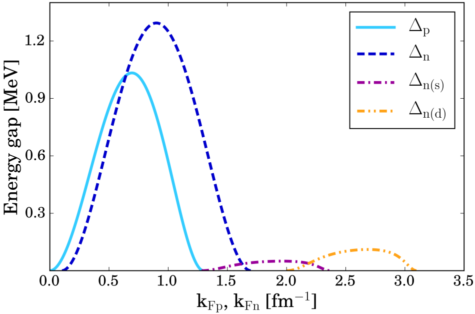


Figure 5: Parametrised energy gaps shown as a function of the proton and neutron Fermi wave numbers, $k_{\rm Fp}$ and $k_{\rm Fn}$, respectively. Singlet-paired gaps for the protons (cyan, solid) and neutrons (blue, dashed) are found on the left. Further on the right, two different neutron triplet gaps are given, i.e. a shallow (purple, dot-dashed) and a deep (yellow, dot-dot-dashed) model.

$$\Delta(k_{\rm Fx}) = \Delta_0 \frac{(k_{\rm Fx} - g_0)^2}{(k_{\rm Fx} - g_0)^2 + g_1} \frac{(k_{\rm Fx} - g_2)^2}{(k_{\rm Fx} - g_2)^2 + g_3}. \tag{18}$$

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