Magnetic field distributions in superconducting neutron stars

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Motivation

Core magnetic field

- Neutron star **magnetic fields** play a **crucial role** in astrophysics, e.g.:
  - explain the high activity of magnetars & high-B field pulsars.
  - field evolution linked to neutron star ‘metamorphosis‘.
  - influence the dynamics of binary neutron star mergers.
  - related to many astrophysical phenomena (e.g., FRBs, GRBs).
- While a lot of progress has been made in modelling the exterior and crustal magnetic fields, there are still **many open questions** regarding the **field in the core**, carrying a significant fraction of the magnetic energy.
- **Open problems concern**, e.g., *correct* multi-fluid description of matter, ambipolar diffusion, role of crust-core interface, evolution time-scales for type-II superconducting matter, and possibility of *fast* core dissipation.

Superconductivity

- The conditions in the interior are such that nucleons can undergo **phase transitions** into **superfluid states** as a result of Cooper pairing. Detailed gap calculations suggest the following **core critical temperatures**:

$$T_{\mathrm{c,\,protons}} \sim 10^9 - 10^{10}\,\mathrm{K}, \qquad T_{\mathrm{c,\,neutrons}} \sim 10^8 - 10^9\,\mathrm{K}, \tag{1}$$

- Our understanding of NS superconductivity is mainly based on **time-independent, single-component** considerations (Baym, Pethick & Pines, 1969):
  - Outer-core protons are in a type-II state with flux confined to a fluxtube array.
  - In the inner core, a transition to an intermediate type-I state takes place.
  - Magnetic flux expulsion times are very long, leading to a meta-stable state.

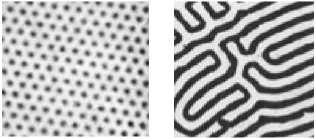


Figure 1: Laboratory type-II and intermediate type-I superconductor (Brandt & Essmann, 1987).

Condensate coupling

What happens to this picture of core superconductivity when coupling (specifically entrainment) between the condensates is included?

- Expanding **earlier works** (Alpar, Langer & Sauls, 1984; Charbonneau & Zhitnitsky, 2007; Alford & Good, 2008; Haber & Schmitt, 2017), we use techniques for laboratory systems to construct **phase diagrams** by deducing the protons' ground state in presence of a magnetic field as a **function of density**.

- With entrainment, **velocity-dependent terms** in energy density read

$$F_{\mathrm{vel}} = \frac{1}{2} m n_{\mathrm{p}} |\mathbf{V}_{\mathrm{p}}|^2 + \frac{1}{2} m n_{\mathrm{n}} |\mathbf{V}_{\mathrm{n}}|^2 - \frac{1}{2} \rho^{\mathrm{pn}} |\mathbf{V}_{\mathrm{p}} - \mathbf{V}_{\mathrm{n}}|^2 , \quad (2)$$

where $n_{\mathrm{p,n}}$ are the nucleon number densities, the coefficient $\rho^{\mathrm{pn}} < 0$ determines the strength of entrainment (Andreev & Bashkin, 1976) and $\mathbf{V}_{\mathrm{p,n}}$ are superfluid velocities related to canonical momenta, i.e., $\propto \nabla \arg \psi_{\mathrm{x}}$.

Ginzburg–Landau model

- In a **mean-field framework**, entrainment first enters at 4th order in $\psi_{\mathrm{n,p}}$ and 2nd order in their derivatives, i.e., we require a linear combination of terms $|\psi_x|^2|\nabla\psi_y|^2, \psi_x\psi_y\nabla\psi_x^\star\cdot\nabla\psi_y^\star, \psi_x\psi_y^\star\nabla\psi_x^\star\cdot\nabla\psi_y, \psi_x^\star\psi_y^\star\nabla\psi_x\cdot\nabla\psi_y$ where $x, y \in \{\mathrm{p}, \mathrm{n}\}$. **Galilean invariance** can be used to simplify the sum.

- The total **free energy density** of our two-component superconductor is

$$F[\psi_\mathsf{p}, \psi_\mathsf{n}, \mathbf{A}] = \frac{1}{8\pi}|\nabla\times\mathbf{A}|^2 + \frac{g_\mathsf{pp}}{2}\left(|\psi_\mathsf{p}|^2 - \frac{n_\mathsf{p}}{2}\right)^2 + \frac{g_\mathsf{nn}}{2}\left(|\psi_\mathsf{n}|^2 - \frac{n_\mathsf{n}}{2}\right)^2$$
$$+ g_\mathsf{pn}\left(|\psi_\mathsf{p}|^2 - \frac{n_\mathsf{p}}{2}\right)\left(|\psi_\mathsf{n}|^2 - \frac{n_\mathsf{n}}{2}\right) + \frac{\hbar^2}{4m_\mathsf{u}}\left|\left(\nabla - \frac{2ie}{\hbar c}\mathbf{A}\right)\psi_\mathsf{p}\right|^2 + \frac{\hbar^2}{4m_\mathsf{u}}|\nabla\psi_\mathsf{n}|^2$$
$$+ h_1\left|\left(\nabla - \frac{2ie}{\hbar c}\mathbf{A}\right)(\psi_\mathsf{n}^\star\psi_\mathsf{p})\right|^2 + \frac{h_2 - h_1}{2}\nabla(|\psi_\mathsf{p}|^2)\cdot\nabla(|\psi_\mathsf{n}|^2)$$
$$+ \frac{h_3}{4}\left(\left|\nabla(|\psi_\mathsf{p}|^2)\right|^2 + \left|\nabla(|\psi_\mathsf{n}|^2)\right|^2\right), \tag{3}$$

where $g_{\mathrm{pp,nn}}$ define the self-repulsion of the condensates, $g_{\mathrm{pn}} \approx 0$ their mutual repulsion and h_i are related to the condensates' coupling.

- To find the **ground state** in the presence of an **imposed magnetic field**, we can control (i) the magnetic flux density, $\mathbf{B} = \nabla \times \mathbf{A}$, by imposing a mean flux $\overline{B}$, or (ii) the thermodynamic external magnetic field $\mathbf{H}$.

- Case (i) approximates the **neutron star interior**, which becomes superconducting as the star cools in the presence of a pre-existing field.

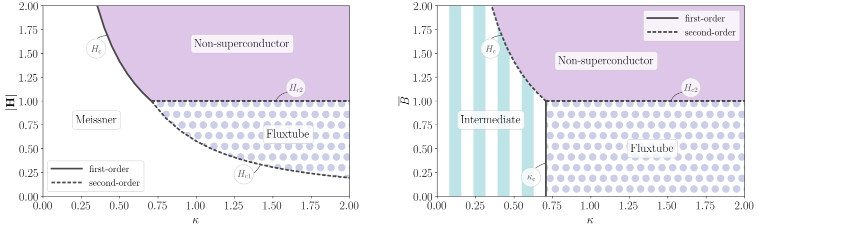


Figure 2: Phase diagrams for a one-component superconductor, for different values of the Ginzburg-Landau parameter, κ. The experiment with an imposed external field, $|\mathbf{H}|$, in nondimensional units is shown on the left, while the right panel shows the phase transitions in the experiment with an imposed mean flux, $\overline{B}$.

Phase diagrams

- For two-component systems, phase diagrams look more complicated and we obtain additional **mixed states** as a result of condensate interactions. These are marked by **first-order transitions** at $H_{c1'} < H_{c1}$ and $H_{c2'} > H_{c2}$.

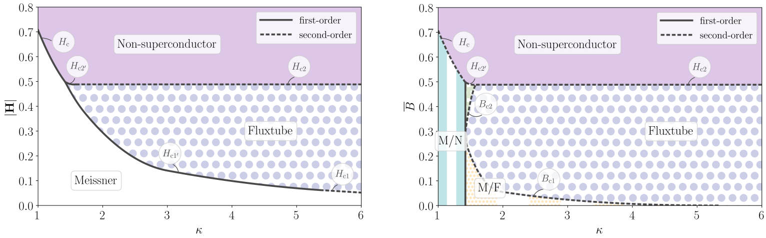
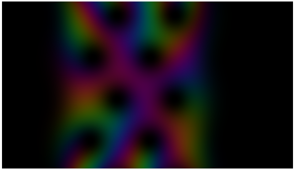


Figure 3: Phase diagrams for our two-component superconductor as a function of κ, and $\sqrt{g_{\mathbf{nn}} n_{\mathbf{n}} / g_{\mathbf{pp}} n_{\mathbf{p}}} = 0.371$, $n_{\mathbf{p}}/n_{\mathbf{n}} = 0.097$, $g_{\mathrm{np}} = 0$, $h_{\mathbf{1}} = 0.102$, $h_{\mathbf{2}} = 0.387$, and $h_{\mathbf{3}} = 0.263$. The shading is indicative of the actual distribution of magnetic flux.

Results

Mixed states

- We find inhomogeneous regimes where fluxtube and non-superconducting regions (left) as well as Meissner and fluxtube regions (right) alternate.



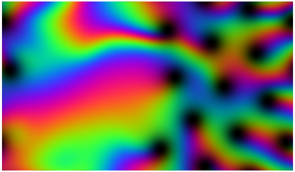


Figure 4: Inhomogeneous ground states, where brightness and hue indicate the density and phase of the proton order parameter, $\psi_{\mathbf{p}}$, respectively.

- Reminiscent of **type-1.5 superconductivity** in terrestrial systems $\Rightarrow$ entrainment causes fluxtube repulsion on short scales & attraction on large scales, resulting in mixed states.

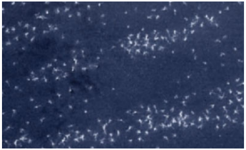


Figure 5: Image of multi-band SC Mg_2B (Moshchalkov et al., 2009).

Results

Skyrme connection

- After determining the **composition** based on the **full Skyrme model**, we link our Ginzburg–Landau model to a reduced Skyrme functional to obtain the coefficients h_i and determine the ground state at different densities.

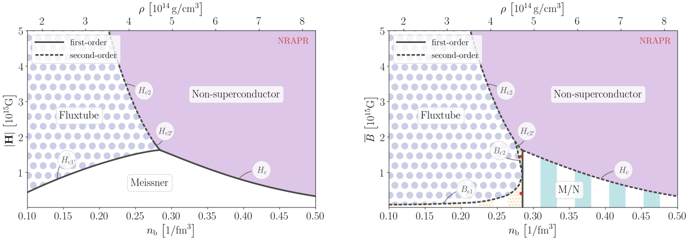
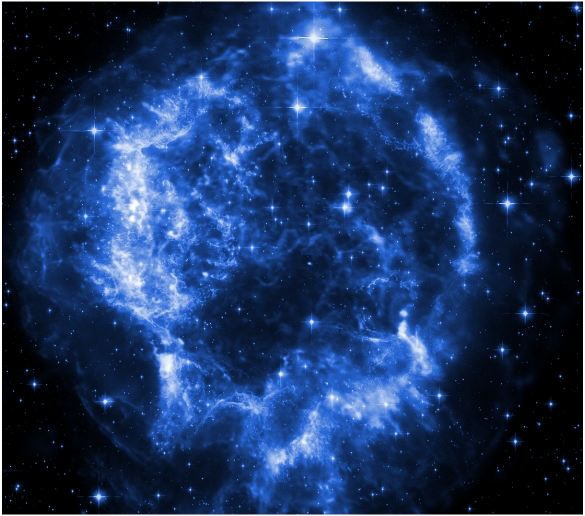


Figure 6: Phase diagrams for the NRAPR equation of state as a function of density.

Although exact positions of transitions are model dependent, for typical EoSs the core retains flux and is occupied by mixed states for fields below $\overline{B} \lesssim 10^{15}\,\mathrm{G}$ and partially by fluxtubes for $10^{15} \lesssim \overline{B} \lesssim 10^{16}\,\mathrm{G}$.



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