Neutron Stars as Cosmic Laboratories

Astrophysics Colloquium



Uni Melbourne Nov. 28, 2018 Vanessa Graber, McGill University vanessa.graber@mcgill.ca





1 Neutron Stars in a Nutshell

- 2 Superfluidity and Superconductivity
- 3 Neutron Star Glitches
- 4 Laboratory Analogues



Contents

1 Neutron Stars in a Nutshell

2 Superfluidity and Superconductivity

3 Neutron Star Glitches

4 Laboratory Analogues

Formation

- Neutron stars are one type of compact remnant, created during the final stages of stellar evolution.
- When a massive star of ~ 8 30 M_☉ runs out of fuel, it collapses under its own gravitational attraction and explodes in a **supernova**.
- During collapse, electron captures $(p + e^- \rightarrow n + \nu_e)$ produce neutrons.
- They have radii between 9 15 kmand weigh $1.2 - 2 M_{\odot}$, resulting in densities up to $\rho \simeq 10^{15} \text{ g cm}^{-3}$.



Figure 1: Snapshot of 3D core-collapse supernova simulation (Mösta et al., 2014).

Structure

- The interior structure is complex and influenced by the (unknown) equation of state. However, there is a canonical understanding.
- After ~ 10⁴ years neutron stars are in equilibrium and have temperatures of 10⁶ - 10⁸ K. They are composed of **distinct layers**.
- For our purposes we separate neutron stars into a solid crust and a fluid core, containing three distinct superfluid components.



Figure 2: Sketch of the neutron star interior.

Quantum condensates

- Neutron stars are hot compared to low-temperature experiments on Earth, but cold in terms of their nuclear physics (Migdal, 1959).
- Neutrons and protons are fermions that can become unstable to Cooper pair formation due to an attractive contribution to the nucleon-nucleon interaction potential.
- Pairing process is described within the standard microscopic BCS theory of superconductivity (Bardeen, Cooper & Schrieffer, 1957).
- Compare the equilibrium to the nucleons' **Fermi temperature**:

$$T_{\rm F} = k_{\rm B}^{-1} E_{\rm F} \sim 10^{12} \,{\rm K} \gg 10^6 - 10^8 \,{\rm K}.$$
 (1)

Neutron star matter is strongly influenced by quantum mechanics!

Transition temperatures

Detailed BCS calculations provide the pairing gaps Δ, which are associated with the critical temperatures T_c for the superfluid and superconducting phase transitions.



Figure 3: Left: Parametrised proton (singlet) and neutron (singlet, triplet) energy gaps as a function of Fermi wave numbers (Ho, Glampedakis & Andersson, 2012). Right: Critical temperatures of superconductivity/superfluidity as a function of the neutron star density. The values are computed for the NRAPR equation of state (Steiner et al., 2005; Chamel, 2008).



Contents



1 Neutron Stars in a Nutshell

2 Superfluidity and Superconductivity

3 Neutron Star Glitches

4 Laboratory Analogues

Basics

- Superfluids flow without viscosity, while superconductors have vanishing electrical conductivity and exhibit Meissner effect.
- Both states involve large numbers of particles condensed into the same quantum state, characteristic for macroscopic quantum phenomena.
- Most of our understanding of superfluidity and superconductivity in neutron stars originates from laboratory counterparts.



Figure 4: Superfluid helium creeps up the walls to eventually empty the bucket.

Superfluid rotation

■ The superfluids can be characterised by macroscopic wave functions Ψ = Ψ₀ e^{iφ} that satisfy the Schrödinger equation. Using the standard formalism one can determine a superfluid velocity

$$\mathbf{v}_{s} \equiv \frac{\mathbf{j}_{s}}{\rho_{s}} = \frac{\hbar}{m_{c}} \nabla \varphi, \qquad \Rightarrow \qquad \boldsymbol{\omega} \equiv \nabla \times \mathbf{v}_{s} = 0.$$
 (2)



Figure 5: Envisage vortices as tiny, rotating tornadoes.

- Superflow is irrotational: the superfluids can only rotate by forming a regular vortex array.
- Each vortex carries a **quantum of circulation** $\kappa = h/2m \approx 2.0 \times 10^{-3} \text{ cm}^2 \text{ s}^{-1}$ and has a size

$$\xi_{\rm v} \approx 1.5 \times 10^{-11} \ (1 - x_{\rm p})^{1/3} \ \left(\frac{m}{m_{\rm n}^*}\right) \ \rho_{14}^{1/3} \ \left(\frac{10^9 \,\rm K}{T_{\rm cn}}\right) \,\rm cm.$$
(3)

Quantised vorticity

The vortices arrange themselves in a hexagonal array (Abrikosov, 1957) and their circulation mimics solid-body rotation on large scales. The averaged vorticity and vortex area density are given by

$$\boldsymbol{\omega} = 2\boldsymbol{\Omega} = \mathcal{N}_{\mathrm{v}}\kappa\hat{\boldsymbol{z}},$$



Figure 6: Vortex array of a rotating superfluid mimics solid-body rotation.

$$\mathcal{N}_{\rm v} \approx 6.3 \times 10^5 \, \left(\frac{10\,{\rm ms}}{P}\right) {\rm cm}^{-2}. \tag{4}$$

■ For a regular array, the intervortex distance is given by d_v ≃ N_v^{-1/2}:

$$d_{\rm v} \approx 1.3 \times 10^{-3} \left(\frac{P}{10 \, {\rm ms}}\right)^{1/2} \, {\rm cm.}$$
 (5)

A change in angular momentum is achieved by creating (spin-up) or destroying (spin-down) vortices.

The vortices interact with the viscous fluid component causing dissipation. This mutual friction influences laboratory systems (Hall & Vinen, 1956) and neutron stars (Alpar, Langer & Sauls, 1984).

Taking $\Omega = \Omega \hat{\Omega}$, the **vortex-averaged** drag force in the core is

$$\boldsymbol{F}_{\rm mf} = 2\mathcal{B}\rho_{\rm n}\,\hat{\boldsymbol{\Omega}} \times [\boldsymbol{\Omega} \times (\boldsymbol{\nu}_{\rm n} - \boldsymbol{\nu}_{\rm e})] + 2\mathcal{B}'\rho_{\rm n}\,\boldsymbol{\Omega} \times (\boldsymbol{\nu}_{\rm n} - \boldsymbol{\nu}_{\rm e})\,. \tag{6}$$

■ The dimensionless parameters *B* and *B*' reflect the strength of *F*_{mf}. They are calculated by considering mesoscopic **coupling physics** for a single vortex and then averaging for the full array.

There are large uncertainties in calculating mutual friction coefficients, which differ between the crust and the core.



Figure 7: Superconducting states.

Type-II state

■ Due to high conductivity, the magnetic flux cannot be expelled from their interiors ⇒ neutron stars do not exhibit Meissner effect and are in a **metastable** state (Baym, Pethick & Pines, 1969; Ho, Andersson & Graber, 2017).

■ The exact phase depends on the **characteristic lengthscales** involved:

$$\kappa = \frac{\lambda}{\xi_{\rm ft}} \approx 3 \left(\frac{m_{\rm p}^*}{m}\right)^{3/2} \rho_{14}^{5/6} \left(\frac{x_{\rm p}}{0.05}\right)^{5/6} \left(\frac{T_{\rm cp}}{10^9 \,\rm K}\right) > \frac{1}{\sqrt{2}}.$$
 (7)

Estimates predict a type-II state in the outer core with

$$H_{c1} = \frac{4\pi \mathcal{E}_{ft}}{\phi_0} \approx 1.9 \times 10^{14} \left(\frac{m}{m_p^*}\right) \rho_{14} \left(\frac{x_p}{0.05}\right) \,\mathrm{G},\tag{8}$$

$$H_{\rm c2} = \frac{\phi_0}{2\pi\xi_{\rm ft}^2} \approx 2.1 \times 10^{15} \left(\frac{m_{\rm p}^*}{m}\right)^2 \rho_{14}^{2/3} \left(\frac{x_{\rm p}}{0.05}\right)^{2/3} \left(\frac{T_{\rm cp}}{10^9 \,\rm K}\right)^2 \,\rm G. \tag{9}$$

Each fluxtube carries a **flux quantum** $\phi_0 = hc/2e \approx 2.1 \times 10^{-7} \,\mathrm{G \, cm}^2$ and has a size

$$\xi_{\rm ft} \approx 3.9 \times 10^{-12} \left(\frac{m}{m_{\rm p}^*}\right) \rho_{14}^{1/3} \left(\frac{x_{\rm p}}{0.05}\right)^{1/3} \left(\frac{10^9 \,{\rm K}}{T_{\rm cp}}\right) \,{\rm cm.} \tag{10}$$

All flux quanta add up to the total magnetic flux. The averaged magnetic induction is related to the fluxtube area density N_{ft}:

$$B = \mathcal{N}_{\rm ft}\phi_0, \quad \rightarrow \quad \mathcal{N}_{\rm ft} \approx 4.8 \times 10^{18} \, \left(\frac{B}{10^{12}\,{
m G}}\right)\,{
m cm}^{-2}.$$
 (11)

The typical **interfluxtube distance** is given by $d_{\rm ft} \simeq \mathcal{N}_{\rm ft}^{-1/2}$ with

$$d_{\rm ft} \approx 4.6 \times 10^{-10} \left(\frac{B}{10^{12} \,{\rm G}}\right)^{-1/2} \,{\rm cm.}$$
 (12)

Field evolution is related to the mechanisms affecting fluxtube motion (Muslimov & Tsygan, 1985; Graber et al., 2015; Graber, 2017, e.g.).

Macroscopic Euler equations for superfluid neutrons and charged fluid in zero-temperature limit (Glampedakis, Andersson & Samuelsson, 2011)

$$\left(\partial_{t}+v_{n}^{j}\nabla_{j}\right)\left[v_{n}^{i}+\varepsilon_{n}w_{np}^{i}\right]+\nabla^{i}\tilde{\Phi}_{n}+\varepsilon_{n}w_{pn}^{j}\nabla^{i}v_{j}^{n}=f_{mf}^{i}+f_{mag,n}^{i},$$
(13)

$$\left(\partial_{t} + v_{p}^{j}\nabla_{j}\right)\left[v_{p}^{i} + \varepsilon_{p}w_{pn}^{j}\right] + \nabla^{i}\tilde{\Phi}_{p} + \varepsilon_{p}w_{np}^{j}\nabla^{i}v_{j}^{p} = -\frac{n_{n}}{n_{p}}f_{mf}^{i} + f_{mag,p}^{i}, \quad (14)$$

with $w_{xy}^i \equiv v_x^i - v_y^i$. Modified by **new force terms**, f_{mf}^i and $f_{mag,x}^i$, due to vortices/fluxtubes and **entrainment**, ε_x (Andreev & Bashkin, 1975).

■ Supplemented by continuity equations and Poisson's equation,

$$\partial_t n_{\mathbf{x}} + \nabla_i (n_{\mathbf{x}} v_{\mathbf{x}}^i) = 0, \qquad \nabla^2 \Phi = 4\pi G \rho,$$
 (15)

and an evolution equation for the magnetic induction B.



Contents



2 Superfluidity and Superconductivity

3 Neutron Star Glitches

4 Laboratory Analogues

Background

Glitches are sudden spin-ups caused by angular momentum transfer from a crustal superfluid, decoupled from the lattice (and everything tightly coupled) due to vortex pinning (Anderson & Itoh, 1975).



Figure 8: Sketch of an idealised glitch.

- Catastrophic vortex unpinning triggers the glitch and frictional forces acting on free vortices govern the neutron star's post-glitch response.
- Observations suggest that the crust spin-up after a glitch is very fast (Dodson, Lewis & McCulloch, 2007; Palfreyman et al., 2018).
- Within hydrodynamical models, the recoupling is captured via the mutual friction coefficient *B*, directly connected to microphysics.

- Decompose the star into **crust superfluid**, **core superfluid** and a non-superfluid **'crust' component**. The latter two rotate rigidly and are coupled via a constant mutual friction $\mathcal{B}_{core} \approx 5 \times 10^{-5}$.
- Neglecting entrainment for simplicity, the equations of motion are

$$\dot{\Omega}_{\rm sf} = \mathcal{B}\left[2\Omega_{\rm sf} + \tilde{r} \frac{\partial\Omega_{\rm sf}}{\partial\tilde{r}}\right] (\Omega_{\rm crust} - \Omega_{\rm sf}), \tag{16}$$

$$\dot{\Omega}_{\rm core} = 2\mathcal{B}_{\rm core}\Omega_{\rm core}(\Omega_{\rm crust} - \Omega_{\rm core}),$$
 (17)

$$\dot{\Omega}_{\rm crust} = -\frac{N_{\rm ext}}{I_{\rm crust}} - \frac{I_{\rm core}}{I_{\rm crust}} \dot{\Omega}_{\rm core} - \frac{1}{I_{\rm crust}} \int \rho \tilde{r}^2 \dot{\Omega}_{\rm sf} \, \mathrm{d}V.$$
(18)

We calculate the coupling B(r̃) in the crust for realistic microphysics and integrate Eqs. (16)-(18) in cylindrical geometry for typical Vela pulsar parameters (Ω_{crust}(0) ≈ 70 Hz, ΔΩ_{crit} ≈ 10⁻² Hz) for 120 s.

Time evolution



Figure 9: Evolution in the inner crust for model (A).

Computing the change in (observed) crust frequency Δν shows that the glitch shape depends crucially on the relative strength between the crust and core mutual friction.

The superfluid rotates **differentially** due to $\mathcal{B}(\tilde{r})$ -dependence. Eventually, it has transferred all excess angular momentum to the crust and spun down to a **new steady state**, where all three components corotate.



Figure 10: Change in crustal frequency with time.

■ First single-pulse observations of a glitch in the Vela pulsar (Palfreyman et al., 2018) allow a **comparison** between the data and our predictions.



Figure 11: Comparison between theoretical timing residuals and observations of the 2016 Vela glitch.

The shape is almost insensitive to the crustal profile as long as $\mathcal{B}\gtrsim 10^{-3}$ but very sensitive to the core coupling. The data suggests a narrow range $3\times 10^{-5}\lesssim \mathcal{B}_{\rm core}\lesssim 10^{-4}$ \Rightarrow test that!!



Contents



2 Superfluidity and Superconductivity

3 Neutron Star Glitches

4 Laboratory Analogues

Objective

- The neutron star interior contains at least three distinct superfluid components and theoretical modelling of their behaviour is very difficult ⇒ use **laboratory counterparts** to understand them better.
- It is not possible to replicate the extreme conditions present in neutron stars. However, we could use known laboratory analogues that are easy to manipulate to recreate and study specific neutron star characteristics.
- I will focus on a few promising examples. For more details see Graber, Andersson & Hogg (2017).



Figure 12: Chandra X-ray observation of the Cassiopeia A supernova remnant.

Helium II spin-up



Figure 13: Schematic setup of the helium II spin-up experiments (Tsakadze & Tsakadze, 1980).

- First (and only) systematic analysis of rotating helium II by Tsakadze & Tsakadze (1980), shortly after first observations of glitches in the Vela and Crab pulsar.
- Validate presence of superfluid components in neutron stars by measuring relaxation timescales after initial changes in the container's rotation.
- Performed for various temperatures, vessel configurations and rotational properties.
- Model comparison is hard (Reisenegger, 1993; van Eysden & Melatos, 2011).

Helium II glitch analogy



Figure 14: Sketch of an idealised neutron star glitch.

Figure 15: Measurement of a laboratory glitch.

- Glitches are not only detected in neutron stars, but have also been observed in laboratory helium experiments. This supports the idea that they are caused by an internal superfluid reservoir.
- There is **only one (!!)** observation of a helium II glitch. Updated experiments could help to understand aspects such as the trigger.

Helium-3

- Helium-3 becomes superfluid below 3 mK. The transition is different to bosonic helium II because helium-3 atoms are fermions and have to form **Cooper-pairs** as expected for the neutron star interior.
- The pairing occurs in a spin-triplet, p-wave state: the Cooper pairs have internal structure resulting in 3 superfluid phases (Vollhardt, 1998).
- The B-phase behaves similar to helium II or the crustal neutron superfluid. The A-phase exhibits anisotropic behaviour and resembles the core neutron superfluid.



Figure 16: Schematic phase diagram of helium-3.

Helium-3 interfaces

■ It is not understood how **interfaces** influence the neutron star dynamics ⇒ **crust-core transition** between two superfluids??



Figure 17: Vortex-line simulation for spin-down of two-phase helium-3 (Walmsley et al., 2011).

Study vortices across an interface with rotating **two-phase samples** (different \mathcal{B} , \mathcal{B}') using NMR measurements and modern vortex-line simulations (Walmsley et al., 2011).

- Interface strongly modifies dynamics:
 - Vortex sheet formation
 - Vortex tangle forms in *B*-phase, reconnections increase dissipation
 - Differential rotation

Interface can become unstable to Kelvin-Helmholtz instability (Finne et al., 2006).

Ultra-cold gases

- A BEC of weakly-interacting bosons was first realised by cooling Rubidium atoms to T ~ nK (Anderson et al., 1995; Davis et al., 1995), and the superfluid transition observed shortly after (Matthews et al., 1999; Madison et al., 2000).
- Although the field is relatively young, ultra-cold gases provide many possibilities to study superfluidity: e.g. fermionic gases, two-component systems, optical lattices, etc.
- Very simple advantage: absorption imaging of clouds is a great tool to study behaviour of individual vortices.



Figure 18: Vortex array in a rotating, dilute BEC of Rubidium atoms (Engels et al., 2002).

GPE modelling



Figure 19: Snapshots of superfluid density during the spin-down of a BEC (Warszawski & Melatos, 2012).

- Time evolution of the **Gross-Pitaevskii equation** describes vortex motion ⇒ use this approach to study the pinned **crustal superfluid** in neutron stars (Warszawski & Melatos, 2012).
- Collective vortex motion in the presence of pinning potential can cause glitch-like events ⇒ study the unknown trigger and glitch statistics.
 - **Two-component GP** formalisms have been used to study neutron star core properties (Alford & Good, 2008; Drummond & Melatos, 2017, e.g.).

Fluxtube physics



Figure 20: 3D STEM tomogram with \sim 70 pinning sites (Ortalan et al., 2009).



Figure 21: Modelled fluxtube motion. Colour reflects order parameter (Sadovskyy et al., 2016).

- Experimental data and modern calculations complement each other: Determine fluxtube motion in a realistic pinning landscape by numerically solving time-dependent Ginzburg-Landau equations.
- Account for pinning defects, fluxtube flexibility, long-range fluxtube repulsion, fluxtube cutting and reconnections.

SC formation



Figure 22: Intermediate state of type-I and type-II phases (Brandt & Essmann, 1987; Essmann, 1971).

- Our understanding of macroscopic superconductivity in neutron stars is based on time-independent equilibrium considerations. It is unclear what happens in detail as the star cools below T_c.
- Experiments could help to better understand the microphysical dynamics of the superconducting phase transition and the resulting flux distribution ⇒ how does the magnetic field actually look like?

Conclusions

- Neutron stars are expected to contain (at least) three distinct quantum condensates that influence the stars' macroscopic behaviour. The majority of our knowledge of these superfluids originates from theoretical work on their laboratory counterparts.
- Neutron star glitches are a direct manifestation of macroscopic superfluidity. Analysing observations of the post-glitch response provides information about the underlying microphysics.
- As significant progress has been made in understanding laboratory condensates, there are many exciting ways to combine both fields of research and probe the dynamics of the neutron star interior with superfluid/superconducting experiments.

Thank you.



Density-dependent \mathcal{B}

- We use the **crustal composition** of Negele & Vautherin (1973) and pinning **interaction parameters** from Epstein & Baym (1992) and Donati & Pizzochero (2006) to calculate B in the inner crust.
- The **bottom of the crust** carries the majority of the crustal mass.



Figure 23: Mutual friction strength for kelvin wave coupling (calculated Epstein & Baym (1992) and Jones (1992) according to as a function of (left) density and (right) relative overlying mass fraction.

GWs from binary NSs

- Superfluidity and superconductivity are usually not accounted for in gravitational wave signal modelling as the effects of macroscopic condensates are generally believed to be negligible.
- It has been suggested that tidal disruption during the late inspiral could dynamically couple to neutron star oscillations. If this is true than superfluidity/superconductivity could modify wave forms.



Figure 24: Time-frequency representation of GW170817 (Abbott et al., 2017).

Quantum states can only be present if stars are cold enough. Not clear how parameters like temperature, conductivities and viscosities evolve during the merger.

GWs from isolated NSs

- Isolated neutron stars are likely to exhibit non-spherical dynamical changes in the interior fluid, which would result in the emission of gravitational waves (small amplitude).
- Interesting oscillations are the r-modes (inertial modes in rotating objects dominated by Coriolis force), because they are susceptible to the CFS (Chandrasekhar-Friedman-Schutz) instability.
- They can be prograde in inertial but retrograde in rotating frame, so that GW emission does not damp but increase amplitudes. Detailed physics will depend on presence of quantum condensates.



Figure 25: Oscillation seen by inertial (left) and rotating (right) observer (animation by Ben Owen).

Type-II/I transition

Figure 26: Density-dependent parameters of NS superconductivity calculated for the NRAPR effective equation of state (Steiner et al., 2005). $T_{\rm CP}$ is obtained from Ho, Glampedakis & Andersson (2012).



- Parameters of superconductivity are dependent on the neutron star density, i.e. the equation of state.
- At higher densities one eventually has κ < 1/√2, so that the type-II state should transition into a type-I state. The critical density is

$$\rho_{\rm crit,II \to I} \approx 6.4 \times 10^{14} \left(\frac{m_{\rm p}^*}{m}\right)^{-\frac{9}{5}} \left(\frac{0.05}{x_{\rm p}}\right) \left(\frac{T_{\rm cp}}{10^9 \,{\rm K}}\right)^{-\frac{6}{5}} {\rm g \, cm^{-3}}.$$
(19)

References I

- Abbott B. P. et al., 2017, PhRvL, 119, 161101
- Abrikosov A. A., 1957, JPCS, 2, 199
- Alford M. G., Good G., 2008, PhRvB, 78, 024510
- Alpar M. A., Langer S. A., Sauls J. A., 1984, ApJ, 282, 533
- Anderson M. H., Ensher J. R., Matthews M. R., Wieman C. E., Cornell E. A., 1995, Sci, 269, 198
- Anderson P. W., Itoh N., 1975, Natur, 256, 25
- Andreev A. F., Bashkin E. P., 1975, JETP, 42, 164
- Bardeen J., Cooper L. N., Schrieffer J. R., 1957, PhRv, 108, 1175
- Baym G., Pethick C. J., Pines D., 1969, Natur, 224, 673
- Brandt E. H., Essmann U., 1987, PSSBR, 144, 13
- Chamel N., 2008, MNRAS, 388, 737
- Davis K., Mewes M.-O., Andrews M. R., van Druten N. J., Durfee D. S., Kurn D. M., Ketterle W., 1995, PhRvL, 75, 3969
- Dodson R., Lewis D., McCulloch P., 2007, Ap&SS, 308, 585
- Donati P., Pizzochero P. M., 2006, PhLB, 640, 74
- Drummond L. V., Melatos A., 2017, MNRAS, 472, 4851
- Engels P., Coddington I., Haljan P. C., Cornell E. A., 2002, PhRvL, 89, 100403
- Epstein R. I., Baym G., 1992, ApJ, 387, 276
- Essmann U., 1971, Phy, 55, 83
- Finne A. P., Eltsov V. B., Hänninen R., Kopnin N. B., Kopu J., Krusius M., Tsubota M., Volovik G. E., 2006, RPPh, 69, 3157

References II

- Graber V., 2017, AN, 338, 1
- Graber V., Andersson N., Glampedakis K., Lander S. K., 2015, MNRAS, 453, 671
- Graber V., Andersson N., Hogg M., 2017, IJMPD, 26, 1730015
- Hall H. E., Vinen W. F., 1956, PSPSA, 238, 215
- Ho W. C. G., Glampedakis K., Andersson N., 2012, MNRAS, 422, 2632
- Ho W. W. C. G., Andersson N., Graber V., 2017, PhRvC, 96, 065801
- Jones P. B., 1992, MNRAS, 257, 501
- Madison K., Chevy F., Wohlleben W., Dalibard J., 2000, PhRvL, 84, 806
- Matthews M. R., Anderson B. P., Haljan P. C., Hall D. S., Wieman C. E., Cornell E. A., 1999, PhRvL, 83, 2498
- Migdal A. B., 1959, NucPh, 13, 655
- Mösta P. et al., 2014, ApJ, 785, L29
- Muslimov A. G., Tsygan A. I., 1985, SvA, 11, 80
- Negele J. W., Vautherin D., 1973, NuPhA, 207, 298
- Ortalan V., Herrera M., Rupich M. W., Browning N. D., 2009, PhyC, 469, 2052
- Palfreyman J., Dickey J. M., Hotan A., Ellingsen S., van Straten W., 2018, Natur, 556, 219
- Reisenegger A., 1993, JLTP, 92, 77
- Sadovskyy I. A., Koshelev A. E., Glatz A., Ortalan V., Rupich M. W., Leroux M., 2016, PhRvP, 5, 014011
- Steiner A. W., Prakash M., Lattimer J. M., Ellis P. J., 2005, PhR, 411, 325
- Tsakadze J. S., Tsakadze S. J., 1980, JLTP, 39, 649
- van Eysden C. A., Melatos A., 2011, JLTP, 165, 1
- Vollhardt D., 1998, in Pair Correlations in Many-Fermion Systems, Kresin V. Z., ed., Springer US, Boston, MA, pp. 205–220
- Walmsley P. M., Eltsov V. B., Heikkinen P. J., Hosio J. J., Hänninen R., Krusius M., 2011, PhRvB, 84, 184532