# Glitch rises as a test for rapid superfluid coupling

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in collaboration with A. Cumming & N. Andersson

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# Background

## Glitches

- **Glitches** are sudden spin-ups caused by angular momentum transfer from a crustal superfluid, decoupled from the lattice (and everything tightly coupled) due to vortex pinning (Anderson and Itoh, 1975).
- Catastrophic vortex unpinning triggers the glitch and frictional forces acting on the **free vortices** govern the **neutron star's post-glitch response**. Observations suggest that **crust spin-up** after a glitch is very fast (Dodson et al., 2007; Palfreyman et al., 2018).
- Within **hydrodynamical models**, this recoupling is captured via a dimensionless **mutual friction** coefficient $\mathcal{B}$. It is directly connected to the **mesoscopic dynamics**, because a single vortex experiences a resistive force $f_{\mathrm{res}} = \rho_{\mathrm{s}} \kappa \mathcal{R} \Delta v$ and $\boldsymbol{\mathcal{B}} = \boldsymbol{\mathcal{R}}/(\mathbf{1} + \boldsymbol{\mathcal{R}}^2)$.

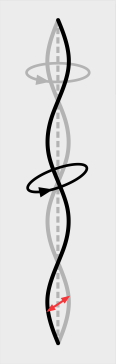
**Learn about the small-scale physics by analysing the glitch rise.**

# Coupling mechanisms

- Different processes affect **vortex dynamics** in the crust and the core:
  - phonon excitations (Jones, 1990)
  - Kelvin wave excitations (Epstein and Baym, 1992; Jones, 1992)
  - electron quasi-particle scattering (Feibelman, 1971)
  - scattering of electrons off the vortex magnetic field (Alpar et al., 1984; Andersson et al., 2006)
  - Kelvin wave excitations (Link, 2003)
- Focus on **Kelvin wave excitations** in the crust, dominating the initial recoupling (if vortex-nucleus velocity is $\Delta v \gtrsim 10^2\,\mathrm{cm\,s^{-1}}$ (Jones, 1992)).
- We reanalyse the works of Epstein and Baym (1992) and Jones (1992), using a **simple argument** to understand discrepancies between both formalisms and **calculate the drag** $\mathcal{R}$ for realistic crust parameters.

# Coupling

# Kelvin waves



- In the absence of forces, a vortex supports **Kelvin waves** with angular frequency $\omega_k = Tk^2/\rho_s\kappa = \hbar k^2/2\mu(k)$ (Thomson, 1880), with tension $T$ and effective mass $\mu(k) \simeq -2m_u/\ln k\xi$.
- Vortex-nucleus **interactions** excite waves with wave numbers $k \lesssim k_* \equiv (2\mu\Delta v/\hbar l)^{1/2} \Rightarrow$ determine the power $p$ transferred into Kelvin waves and relate it to the **resistive force**, $f_{res} = p/\Delta v$.
- Epstein & Baym and Jones make **different assumptions** about $p$ and the interaction potential including the typical interaction scale $l$:

$$E_{EB}(s) = \frac{E_s}{(1+s^2/R_N^2)^4} + \frac{E_l}{1+s^2/R_N^2}, \qquad E_J(s) = E_p \exp\left(-\frac{s^2}{2\xi^2}\right), \qquad (1)$$

where $s$ is the separation, $E_s$ ($E_l$) a short-range (long-range) interaction contribution, $R_N$ the nuclear radius and $\xi$ the coherence length.

# Coupling

## Drag coefficients

- Drag coefficients depend on the relative **vortex-nucleus velocity**, but $E_\mathrm{p}$ and $\Delta v$ are connected by a mesoscopic force balance, $\Delta v \simeq f_\mathrm{pin}/\rho_\mathrm{s}\kappa \sim E_\mathrm{p}/la\rho_\mathrm{s}\kappa$, for a pinning force $f_\mathrm{pin}$ per unit length and lattice spacing $a$.

- Correcting several errors in Epstein and Baym (1992) and accounting for a **reduction factor** $\delta$ due to averaging the microscopic pinning interaction over a mesoscopic vortex length scale (Seveso et al., 2016), we find

$$\boxed{\mathcal{R}_\mathrm{EB} \simeq 2.8 \left(\frac{\mu}{\hbar}\right)^{1/2} \left(\frac{E_\mathrm{p}\delta}{\rho_\mathrm{s}\kappa}\right)^{1/2} \frac{R_\mathrm{N}}{a^{3/2}},} \qquad \boxed{\mathcal{R}_\mathrm{J} \simeq \frac{1}{2\sqrt{\pi}} \left(\frac{\mu}{\hbar}\right)^{1/2} \left(\frac{E_\mathrm{p}\delta}{\rho_\mathrm{s}\kappa}\right)^{1/2} \frac{a^{1/2}}{\xi},} \tag{2}$$

with $E_\mathrm{p}^2 \simeq E_\mathrm{s}^2 + E_\mathrm{l}E_\mathrm{s} + 0.5E_\mathrm{l}^2$ in Epstein and Baym's formalism.

**Calculate $\mathcal{B} \simeq \mathcal{R}$ for a realistic crust model based on three different microscopic models.**

# Coupling

## Density-dependence

- We use the **crustal composition** of Negele and Vautherin (1973) and pinning **interaction parameters** from Epstein and Baym (1992) and Donati and Pizzochero (2006) to calculate $\mathcal{R}$ in the inner crust.
- The **bottom of the crust** carries the majority of the crustal mass.

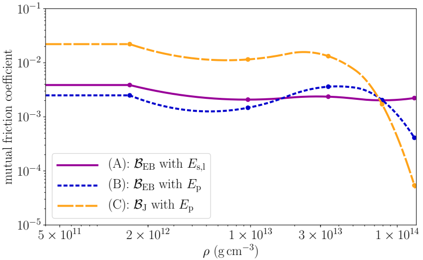



**Figure 1:** Mutual friction strength for kelvin wave coupling as a function of (left) density and (right) relative overlying mass fraction.

# Model

## Three components

- Decompose the neutron star into crust superfluid, core superfluid and a non-superfluid 'crust' component. The latter two rotate rigidly and are coupled via a constant mutual friction coefficient $\mathcal{B}_{\rm core}$.
- Neglecting entrainment for simplicity, the **equations of motion** are

$$\dot{\Omega}_{\rm sf} = \mathcal{B} \left[ 2\Omega_{\rm sf} + \tilde{r}\, \frac{\partial \Omega_{\rm sf}}{\partial \tilde{r}} \right] (\Omega_{\rm crust} - \Omega_{\rm sf}), \tag{3}$$

$$\dot{\Omega}_{\rm core} = 2\mathcal{B}_{\rm core}\Omega_{\rm core}\, (\Omega_{\rm crust} - \Omega_{\rm core}), \tag{4}$$

$$\dot{\Omega}_{\rm crust} = -\frac{N_{\rm ext}}{I_{\rm crust}} - \frac{I_{\rm core}}{I_{\rm crust}}\, \dot{\Omega}_{\rm core} - \frac{1}{I_{\rm crust}} \int \rho \tilde{r}^2 \dot{\Omega}_{\rm sf}\, {\rm d}V. \tag{5}$$

- Relate $\rho$ and $\tilde{r}$ in the crust by solving the **TOV equations** for a realistic EoS to obtain $\mathcal{B}(\tilde{r})$ and integrate (3)-(5) in cylindrical geometry for **Vela pulsar** ($\Omega_{\rm crust}(0) \approx 70\,{\rm Hz}$, $\Delta\Omega_{\rm crit} \approx 10^{-2}\,{\rm Hz}$).

# Differential rotation

- Solve equations for two **fiducial crust-core couplings**: $\mathcal{B}_{\rm core} \sim 5 \times 10^{-5}$ (electron-vortex scattering) and $\mathcal{B}_{\rm core} \sim 10^{-2}$ (Kelvin wave excitations).
- The superfluid rotates **differentially** due to the $\mathcal{B}(\tilde{r})$-dependence. In the outer layers, $\mathcal{B}$ is strongest and the superfluid couples within $\sim 100\,\mathrm{ms}$. Eventually, the superfluid has transferred all excess angular momentum and spun down to a **new steady state**, where all components corotate.

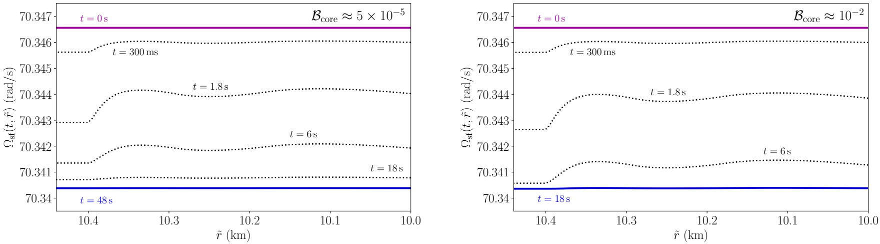



**Figure 2:** $\Omega_{\rm sf}$ as function of radius and time calculated for drag profile (A) and two crust-core couplings.

# Crustal evolution I

- We compare different **friction profiles** by computing the change in crust frequency $\Delta\nu$. The glitch rise shape depends crucially on the **relative strength** of the crust and core mutual friction.
- For $\mathcal{B}_{\rm core} \sim 5 \times 10^{-5}$: crustal coupling is faster than core coupling, creating a characteristic **overshoot** feature. The onset of crust-core coupling is visible as a break in the phase shift $\phi$.

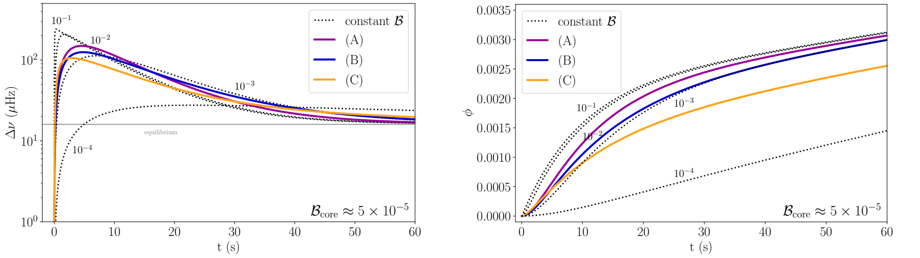



**Figure 3:** Change in crustal frequency $\Delta\nu(t) = [\Omega_{\rm crust}(t) - \Omega_{\rm crust}(0)]/2\pi$ and phase shift $\phi = \int \Delta\nu \, {\rm d}t$ with time.

# Crustal evolution II

- For $\mathcal{B}_{\rm core} \sim 10^{-2}$: crustal coupling is slower than core coupling, causing the glitch rise to be **monotonic in time**. The onset of crust-core coupling is not visible in the **phase shift** $\phi$.

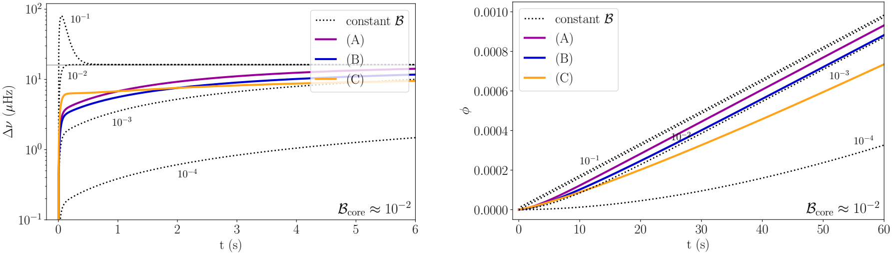



**Figure 4:** Change in crustal frequency $\Delta\nu(t) = [\Omega_{\rm crust}(t) - \Omega_{\rm crust}(0)]/2\pi$ and phase shift $\phi = \int \Delta\nu \, {\rm d}t$ with time.

**Detecting a break in phase shift allows us to constrain $\mathcal{B}_{\rm core}$.**

# Data comparison I

- First single-pulse observations of a glitch in the Vela pulsar (Palfreyman et al., 2018) allow a **comparison** between the data and our predictions.
- Model **timing residuals** $\Delta t \simeq -2\pi\phi/\Omega_{\text{crust}}(0)$ are compared to observed residuals. We include a shift $\Delta t_0 \approx 0.22\,\mathrm{ms}$ at the time of the glitch.

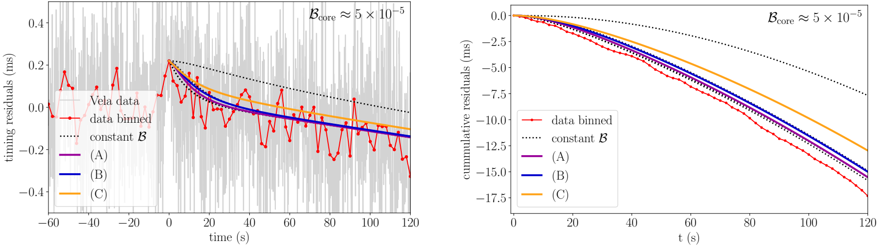



**Figure 5:** Comparison between theoretical (left) timing residuals and (right) cumulative residuals for $\mathcal{B}_{\text{core}} \approx 5 \times 10^{-5}$ and observations of the 2016 Vela glitch.

**Shape is rather insensitive to crustal profiles as long as $\mathcal{B} \gtrsim 10^{-3}$.**

# Data comparison II

- Analyse how sensitive the glitch rise is to $\mathcal{B}_{\rm core}$ for crustal profile (A):

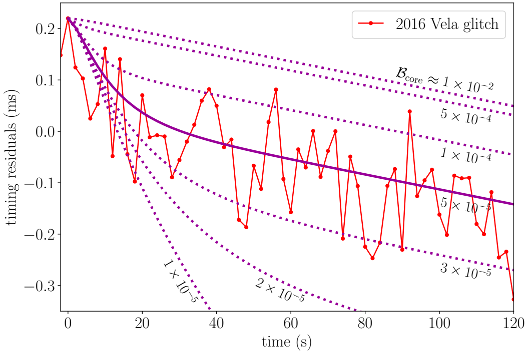



**Figure 6:** Comparison between the 2016 Vela glitch data and theoretical predictions calculated for drag profile (A) and a varying crust-core mutual friction strength $\mathcal{B}_{\rm core}$.

**The data would suggest a narrow range $3 \times 10^{-5} \lesssim \mathcal{B}_{\rm core} \lesssim 10^{-4}$.**

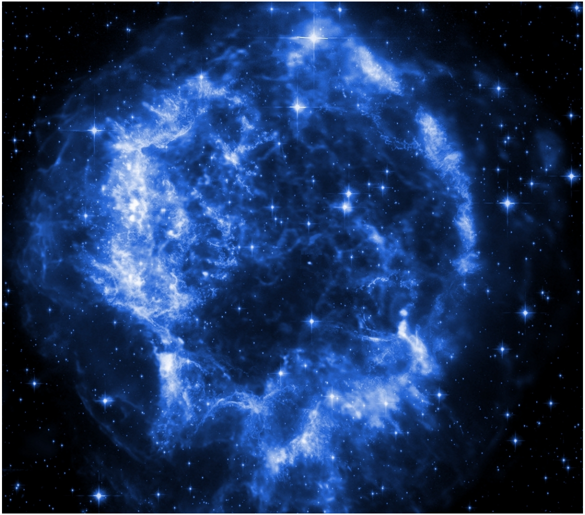
# Conclusions

- In order to **constrain microphysics** of neutron stars, we need to understand the connection between the macroscopic observables and microphysics $\Rightarrow$ develop a predictive **glitch rise** model.

- Combine realistic **Kelvin-wave profiles** in the crust with a simple treatment of the core mutual friction and implement both in a **three-component** neutron star framework $\Rightarrow$ glitch shape depends crucially on the **relative strength** between $\mathcal{B}$ and $\mathcal{B}_{\rm core}$.

- **Comparison** between our models and the first pulse-to-pulse **glitch observations** suggest strong crustal combined with weak core mutual frictional $\Rightarrow$ i.e. $\boldsymbol{\mathcal{B} \gtrsim 10^{-3}}$ and $\boldsymbol{3 \times 10^{-5} \lesssim \mathcal{B}_{\rm core} \lesssim 10^{-4}}$.

# The end



## References I


Alpar, M. A., Langer, S. A., and Sauls, J. A. (1984). Rapid postglitch spin-up of the superfluid core in pulsars. *ApJ*, 282:533–541.

Anderson, P. W. and Itoh, N. (1975). Pulsar glitches and restlessness as a hard superfluidity phenomenon. *Natur*, 256(5512):25–27.

Andersson, N., Sidery, T., and Comer, G. L. (2006). Mutual friction in superfluid neutron stars. *MNRAS*, 368(1):162–170.

Chamel, N. (2008). Two-fluid models of superfluid neutron star cores. *MNRAS*, 388(2):737–752.

Dodson, R., Lewis, D., and McCulloch, P. (2007). Two decades of pulsar timing of Vela. *Ap&SS*, 308(1-4):585–589.

Donati, P. and Pizzochero, P. M. (2006). Realistic energies for vortex pinning in intermediate-density neutron star matter. *PhLB*, 640(3):74–81.

Epstein, R. I. and Baym, G. (1992). Vortex drag and the spin-up time scale for pulsar glitches. *ApJ*, 387:276–287.

Feibelman, P. J. (1971). Relaxation of Electron Velocity in a Rotating Neutron Superfluid: Application to the Relaxation of a Pulsar's Slowdown Rate. *PhRvD*, 4(6):1589–1597.

Ho, W. C. G., Glampedakis, K., and Andersson, N. (2012). Magnetars: super(ficially) hot and super(fluid) cool. *MNRAS*, 422(3):2632–2641.

Jones, P. B. (1990). Rotation of the neutron-drip superfluid in pulsars: The resistive force. *MNRAS*, 243:257–262.

Jones, P. B. (1992). Rotation of the neutron-drip superfluid in pulsars: The Kelvin phonon contribution to dissipation. *MNRAS*, 257(3):501–506.

Link, B. (2003). Constraining Hadronic Superfluidity with Neutron Star Precession. *PhRvL*, 91(10):101101.

# References II

Negele, J. W. and Vautherin, D. (1973). Neutron star matter at sub-nuclear densities. *NuPhA*, 207(2):298–320.

Palfreyman, J., Dickey, J. M., Hotan, A., Ellingsen, S., and van Straten, W. (2018). Alteration of the magnetosphere of the Vela pulsar during a glitch. *Natur*, 556(7700):219–222.

Seveso, S., Pizzochero, P. M., Grill, F., and Haskell, B. (2016). Mesoscopic pinning forces in neutron star crusts. *MNRAS*, 455(4):3952–3967.

Steiner, A. W., Prakash, M., Lattimer, J. M., and Ellis, P. J. (2005). Isospin asymmetry in nuclei and neutron stars. *PhR*, 411(6):325–375.

Thomson, W. (1880). Vibrations of a columnar vortex. *PMag*, 10(61):155–168.