Mutual friction in neutron star crusts

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Background

Superfluid components

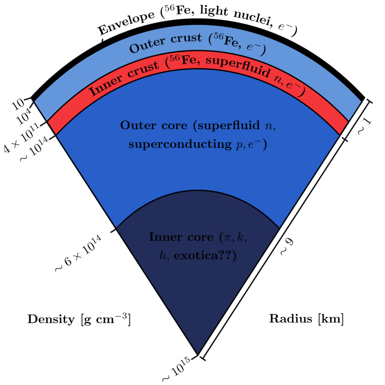


Figure 1: Layered neutron star structure.

- Equilibrium neutron stars with $10^6 - 10^8$ K are cold enough to contain **superfluid** neutrons and **superconducting** protons.
- Cooper pair formation occurs due to an **attractive contribution** to the nucleon-nucleon interaction.
- In the inner crust, neutrons undergo $^1S_0$ pairing with $\boldsymbol{T_c} \sim 10^9 - 10^{10}$ K (Gezerlis, Pethick & Schwenk, 2014).

This crustal superfluid can affect macroscopic observables.

Glitches

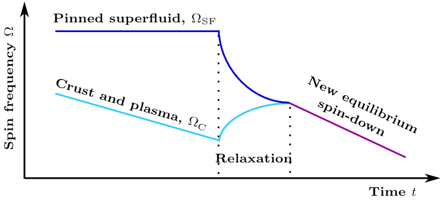


Figure 2: Sketch of an idealised neutron star glitch.

- Glitches are **sudden spin-ups** that interrupt the regular spin-down behaviour of pulsars. The phenomenon is generally attributed to internal dynamics, supported by lab experiments (Tsakadze & Tsakadze, 1980).

- Vela-type glitches with $\Delta\Omega/\Omega \sim 10^{-6}$ are thought to be caused by **angular momentum transfer** from a faster rotating superfluid. Its spin-down is impeded due to vortex pinning (Haskell & Melatos, 2015).

- The **core superfluid** is strongly coupled to the crust with $t_{\mathrm{cpl}} \sim 10P$ and does not participate in the glitch (Alpar, Langer & Sauls, 1984).

Two-component model

- The superfluid involved in the glitch process contains only a few % of the star's total angular momentum (Link, Epstein & Lattimer, 1999).
  $\Rightarrow$ the **crustal superfluid** is responsible for the glitch.
- The **post-glitch recovery** is thus caused by the gradual repinning of vortices (Pines & Alpar, 1985) and characterised by very long relaxation timescales on the order of weeks to years (Shannon et al., 2016, e.g.).
- Glitch dynamics are usually modelled by a simple **two-component model**: superfluid plus crust (and everything tightly coupled to it):

$$I_{\rm c}\dot{\Omega}_{\rm c} = N_{\rm ext} - N_{\rm int}, \qquad I_{\rm sf}\dot{\Omega}_{\rm sf} = N_{\rm int}. \tag{1}$$

$N_{\rm int}$ depend crucially on vortex interactions. In hydrodynamical models this is captured by **mutual friction coefficients** giving $t_{\rm cpl} \sim 1/2\Omega\mathcal{B}$.

Composition I

- Vortex dynamics in the crust are poorly understood and **several interaction mechanisms** have been studied in the literature.
- Glitch models neglect that the mutual friction coefficients are not constant but vary with density (Haskell, Pizzochero & Sidery, 2012, e.g.).

We consider three processes for **realistic microscopic parameters**.

- Use the crustal composition calculated by Negele & Vautherin (1973), which is based on the **Wigner-Seitz approximation**.
- The crust is decomposed into identical, spherical cells containing a lattice nucleus surrounded by a free neutron gas and rel. electrons.

Microphysics

Composition II

	I	II	III	IV	V
n_b $[10^{-4}\,\mathrm{fm}^{-3}]$	8.8	57.7	204.0	475.0	789.0
Z	40	50	50	40	32
N	280	1050	1750	1460	950
$\tilde{x}$	0.53	0.45	0.35	0.28	0.16
n_G $[10^{-4}\,\mathrm{fm}^{-3}]$	4.8	47.0	184.0	436.0	737.0
ρ $[10^{12}\,\mathrm{g\,cm}^{-3}]$	1.5	9.6	33.9	78.9	131.0
A	115	161	193	183	232
R_WS [fm]	44.3	35.7	27.6	19.6	14.4
n_l $[10^{-6}\,\mathrm{fm}^{-3}]$	2.7	5.2	11.3	31.7	80.3
a [fm]	90.0	72.5	56.1	39.8	29.2
c_s $[10^{8}\,\mathrm{cm\,s}^{-1}]$	4.73	5.57	5.78	5.64	4.68
n_e $[10^{-4}\,\mathrm{fm}^{-3}]$	1.10	2.62	5.67	12.67	25.71

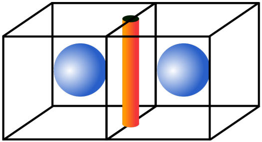
Table 1: Equilibrium composition for five crustal regions. Baryon density n_b, proton Z and neutron number N within a Wigner-Seitz sphere, proton-to-neutron ratio $\tilde{x}$ inside a nucleus and free neutron density n_G are taken from Negele & Vautherin (1973). Total mass density ρ, number of baryons inside a nucleus A, Wigner-Seitz radius R_WS, density of lattice sites n_l, bcc lattice constant a, phonon velocity c_s and electron density n_e are calculated.

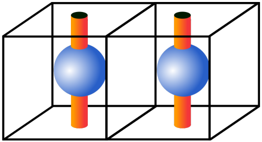
Microphysics

Pinning characteristics I

- Coupling between **vortices** and **crustal lattice** is influenced by gap Δ, coherence length ξ, pinning energy E_{p} and radius R_{N} of nuclei.
- Vortex-nucleus interaction and the resulting pinning phenomenon are difficult to model. We use the results from Donati & Pizzochero (2006) based on a **semi-classical local density approximation**.
- The pinning energy per pinning site represents **energy gain/loss** for positioning a single lattice nucleus inside a vortex. Depending on the density **different pinning configurations** are possible:

interstitial pinning $E_{\mathrm{p}} > 0$





nuclear pinning $E_{\mathrm{p}} < 0$

Microphysics

Pinning characteristics II

	I	II	III	IV	V
Δ [MeV]	0.21	0.68	0.91	0.55	0.19
ξ [fm]	20.0	13.0	15.4	33.5	116.4
$E_{\rm p}$ [MeV]	0.21	0.29	-2.74	-0.72	-0.02
$R_{\rm N}$ [fm]	6.0	6.7	7.3	6.7	5.2

Table 2: Neutron pairing gap Δ, coherence length ξ, microscopic pinning energy per nucleus $E_{\rm p}$ and radius of lattice nuclei $R_{\rm N}$ for five crustal regions from Donati & Pizzochero (2006).

- Macroscopic dynamics of the superfluid are not influenced by the microscopic $E_{\rm p}$, but the **pinning energy per unit length** of vortex.
- **Vortex rigidity** affects averaged pinning force. A vortex is straight over a length scale of order $\sim 10^3 R_{\rm WS}$ (Grill & Pizzochero, 2012).
- Averaging over this scale leads to a reduction in the pinning energy by two orders of magnitude (Seveso et al., 2016). Follow Jones (1992) and account for this by introducing a **reduction factor** $\delta \sim 10^{-4}$.

Coupling

Phonon excitation

- Vortex motion past the crustal lattice causes friction dependent on their **relative velocity**. For low $|\Delta \boldsymbol{v}| \equiv |\boldsymbol{v}_\mathrm{l} - \boldsymbol{u}_\mathrm{v}| \leq 10^2\,\mathrm{cm\,s^{-1}}$, the interaction is dominated by **phonon excitations** (Jones, 1990, 1992).

- A straight vortex segment feels a **resistive force** per unit length

$$\mathbf{f}_\mathrm{ph} = \gamma_\mathrm{ph}\,(\boldsymbol{v}_\mathrm{l} - \boldsymbol{u}_\mathrm{v}) = m n_\mathrm{G} \kappa \mathcal{R}_\mathrm{ph}\,(\boldsymbol{v}_\mathrm{l} - \boldsymbol{u}_\mathrm{v}). \tag{2}$$

- Jones (1990) calculates the **drag coefficient**, which determines the dimensionless **mutual friction parameter** (weak limit $\mathcal{R}_\mathrm{ph} \ll 1$)

$$\mathcal{B}_\mathrm{ph} \simeq \mathcal{R}_\mathrm{ph} = \frac{3}{32\pi^{1/2}\kappa m^2}\,\frac{a}{\xi^3}\,\frac{E_\mathrm{p}^2\delta}{A n_\mathrm{G} c_\mathrm{s}^3} \approx 2.4\times 10^{-9}\left(\frac{a}{60\,\mathrm{fm}}\right)\left(\frac{15\,\mathrm{fm}}{\xi}\right)^3$$

$$\times\left(\frac{|E_\mathrm{p}|}{1\,\mathrm{MeV}}\right)^2\left(\frac{\delta}{10^{-4}}\right)\left(\frac{180}{A}\right)\left(\frac{10^{-2}\,\mathrm{fm}^{-3}}{n_\mathrm{G}}\right)\left(\frac{10^9\,\mathrm{cm\,s^{-1}}}{c_\mathrm{s}}\right)^3. \tag{3}$$

Coupling | Kelvin wave excitation I

- For large $|\Delta \boldsymbol{v}| \geq 10^2\,\mathrm{cm\,s^{-1}}$ vortices oscillate, which excites **Kelvin waves** along their axes and destroys their rigidity (Thomson, 1880).

- Epstein & Baym (1992) consider energy loss due to the excitation of unperturbed Kelvin modes and find **velocity-dependent drag** with

$$\mathcal{R}_{\mathrm{kel}} = \left(\frac{v_*}{|\Delta \boldsymbol{v}|}\right)^{\frac{3}{2}}, \quad \text{where} \quad v_* \equiv 1.1\, f(E_{\mathrm{p}}) \left(\frac{1}{\hbar \kappa^4 m^3} \frac{n_{\mathrm{l}}^2}{R_{\mathrm{N}}\, n_{\mathrm{G}}^4}\right)^{\frac{1}{3}} \tag{4}$$

is a **characteristic velocity** that can be estimated as

$$v_* \approx 2.1 \times 10^6 \left(\frac{1.83}{f(1\,\mathrm{MeV})}\right) \left(\frac{n_{\mathrm{l}}}{10^{-5}\,\mathrm{fm}^{-3}}\right)^{\frac{2}{3}} \left(\frac{7\,\mathrm{fm}}{R_{\mathrm{N}}}\right)^{\frac{1}{3}} \left(\frac{10^{-2}\,\mathrm{fm}^{-3}}{n_{\mathrm{G}}}\right)^{\frac{4}{3}} \frac{\mathrm{cm}}{\mathrm{s}}. \tag{5}$$

- For typical parameters and accounting for δ ($\mathcal{R}_{\mathrm{kel}} \propto E_{\mathrm{p}}^2$), we find

$$\mathcal{B}_{\mathrm{kel}} \simeq \mathcal{R}_{\mathrm{kel}} \approx 0.1 \left(\frac{v_*}{10^6\,\mathrm{cm\,s^{-1}}}\right)^{\frac{3}{2}} \left(\frac{10^4\,\mathrm{cm\,s^{-1}}}{|\Delta \boldsymbol{v}|}\right)^{\frac{3}{2}} \left(\frac{\delta}{10^{-4}}\right). \tag{6}$$

Kelvin wave excitation II

- Kelvin excitation was also studied by Jones (1992). With a different approach, he finds an **energy transfer rate** characterised by

$$\mathcal{R}_{\rm kel} = \frac{E_{\rm p}^2}{\kappa^2 m^2 a n_{\rm G}^2 \xi^2} \left(\frac{1}{6\kappa\xi|\Delta \boldsymbol{v}|^3} \right)^{\frac{1}{2}} . \tag{7}$$

- Using typical parameters and accounting for δ, we recover again a **weak mutual friction limit**

$$\mathcal{B}_{\rm kel} \simeq \mathcal{R}_{\rm kel} \approx 0.1 \left(\frac{|E_{\rm p}|}{1\,{\rm MeV}} \right)^2 \left(\frac{\delta}{10^{-4}} \right) \left(\frac{60\,{\rm fm}}{a} \right) \times \left(\frac{15\,{\rm fm}}{\xi} \right)^{\frac{5}{2}} \left(\frac{10^{-2}\,{\rm fm}^{-3}}{n_{\rm G}} \right)^2 \left(\frac{10^4\,{\rm cm\,s}^{-1}}{|\Delta \boldsymbol{v}|} \right)^{\frac{3}{2}} . \tag{8}$$

- Both methods recover the same $|\Delta \boldsymbol{v}|$-behaviour but show **different dependence** on the microscopic parameters, specifically $E_{\rm p}$.

Electron contribution

- Feibelman (1971) studied **electron scattering** off quasi-particles thermally excited inside the vortex cores. This is strongly dependent on temperature and superfluid parameters and suppressed below $T_{\rm c}$.
- Solving the Boltzman equation, he finds the **relaxation timescale**

$$\tau_{\rm v} \approx 6.3 \times 10^2 \frac{1}{\mathcal{N}\xi^2} \frac{\hbar c\, n_{\rm e}^{2/3}}{\Delta n_{\rm G}^{1/3}} \frac{\hbar}{k_{\rm B} T} \,\mathrm{Exp}\left[\frac{1}{2(9\pi)^{1/3}} \frac{m}{\hbar^2 n_{\rm G}^{2/3}} \frac{\Delta^2}{k_{\rm B} T}\right]. \quad (9)$$

- A vortex experiences the following **drag force per unit length**

$$\boldsymbol{f}_{\rm e} = m_{\rm e} n_{\rm e} \mathcal{N}^{-1} \tau_{\rm v}^{-1} (\boldsymbol{v}_{\rm e} - \boldsymbol{u}_{\rm v}) = m n_{\rm G} \kappa \mathcal{R}_{\rm e} (\boldsymbol{v}_{\rm e} - \boldsymbol{u}_{\rm v}), \quad (10)$$

which gives for the **mutual friction coefficient** (at $T \sim 10^9$ K):

$$\mathcal{B}_{\rm e} \simeq \mathcal{R}_{\rm e} \approx 4.4 \times 10^{-10} \left(\frac{n_{\rm e}}{10^{-4}\,\mathrm{fm}^{-3}}\right) \left(\frac{10^{-2}\,\mathrm{fm}^{-3}}{n_{\rm G}}\right) \left(\frac{P}{10\,\mathrm{ms}}\right) \left(\frac{10\,\mathrm{s}}{\tau_{\rm v}}\right). \quad (11)$$

Mutual friction cross-section

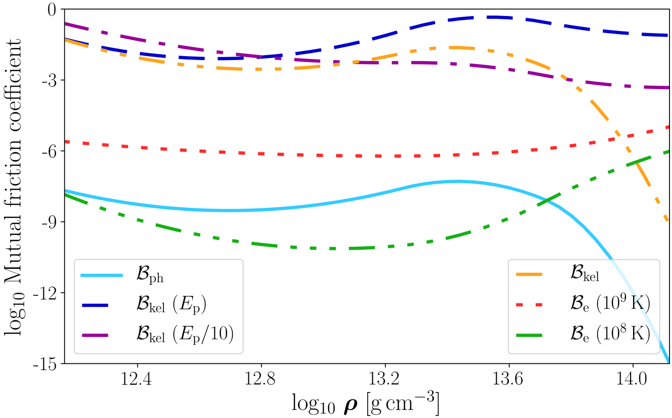


Figure 3: Crustal mutual friction coefficients for phonon coupling, kelvin wave excitations and electron-vortex scattering. The estimates are based on the EoS by Negele & Vautherin (1973), the superfluid parameters of Donati & Pizzochero (2006) and Epstein & Baym (1992), a reduction factor of $\delta = 10^{-4}$ and a relative vortex-lattice velocity of $|\Delta \boldsymbol{v}| = 6.3 \times 10^4$ cm s^{-1}.

Coupling

Mutual friction cross-section

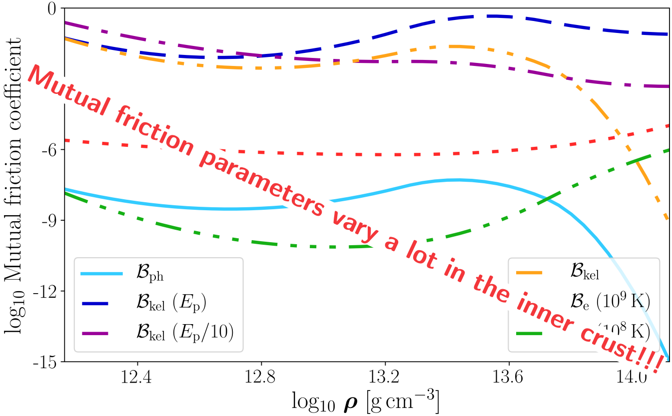


Figure 4: Crustal mutual friction coefficients for phonon coupling, kelvin wave excitations and electron-vortex scattering. The estimates are based on the EoS by Negele & Vautherin (1973), the superfluid parameters of Donati & Pizzochero (2006) and Epstein & Baym (1992), a reduction factor of $\delta = 10^{-4}$ and a relative vortex-lattice velocity of $|\Delta \boldsymbol{v}| = 6.3 \times 10^4$ cm s^{-1}.

Relative velocity

- $|\Delta \boldsymbol{v}|$ is difficult to determine, but body-averaged glitch models provide an estimate: unpinned vortices are locally comoving with the superfluid as a result of the **Magnus force**.
- Approximate $|\Delta \boldsymbol{v}|$ as the maximum velocity lag Δv_{max} between the superfluid and the crust, estimated as (Epstein & Link, 2000)

$$\Delta v_{\mathrm{max}} \simeq |\dot{\Omega}| t_{\mathrm{glitch}} R \tag{12}$$

  with spin-down rate $|\dot{\Omega}|$, interglitch time t_{glitch} and radius R.
- For the **Vela pulsar**, one can estimate with $R = 10\,\mathrm{km}$, $t_{\mathrm{glitch}} \approx 2\,\mathrm{yr}$ and $|\dot{\Omega}| \approx 10^{-10}\,\mathrm{s}^{-2}$ (Dodson, McCulloch & Lewis, 2002)

$$|\Delta \boldsymbol{v}| \approx 6.3 \times 10^{4}\,\mathrm{cm\,s}^{-1}. \tag{13}$$

Glitches in 4He

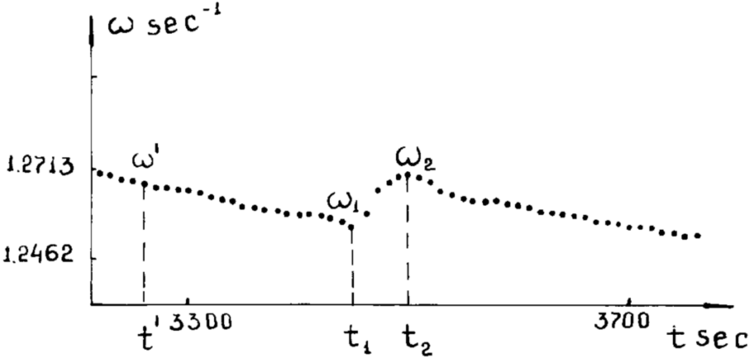


Figure 5: Measurement of a laboratory glitch by Tsakadze & Tsakadze (1980).

Analogy to two-phase 3He

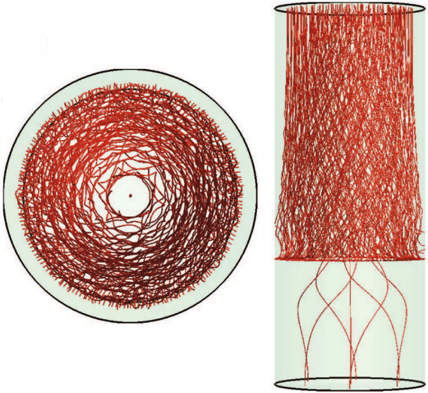


Figure 6: Vortex-line simulation for the spin-down behaviour of a two-phase helium-3 sample. Due to differences in mutual friction, the phases react differently and start to exhibit unusual vortex behaviour (Walmsley et al., 2011).

Core mutual friction

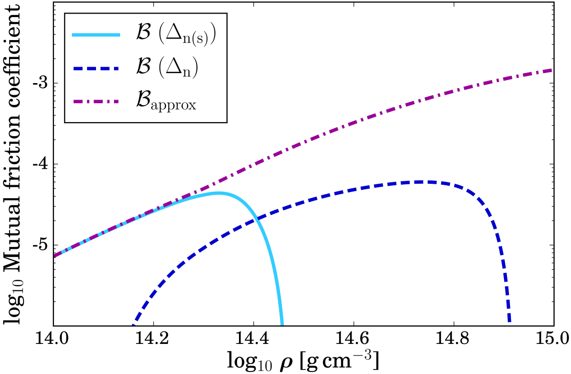


Figure 7: Mutual friction strength in the neutron star core resulting from the scattering of electrons off the vortex magnetic field. The field is generated by the entrained proton flow. Values are calculated for the NRAPR EoS (Steiner et al., 2005) and parameterised superfluid gaps (Ho, Glampedakis & Andersson, 2012).

Snowplough glitch model

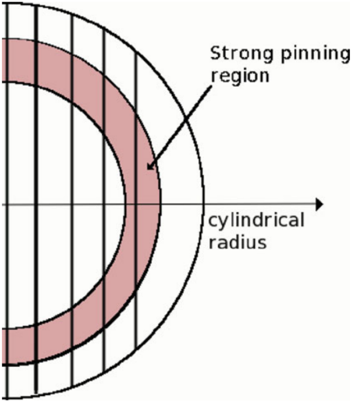


Figure 8: Schematic representation of snowplough glitch model. Continuous vortices thread the entire star, are free to move in the core and experience strong pinning in the inner crust. The larger the vortex segment immersed into the pinning region, the stronger the total pinning force, which has a maximum at large radii in the equatorial plane (Pizzochero, 2011; Haskell, Pizzochero & Sidery, 2012).

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