Superconducting Phases in Neutron Star Cores

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Three condensates

- Detailed BCS calculations provide the pairing gaps Δ, which are associated with the **critical temperatures** T_c for the superfluid and superconducting phase transitions.

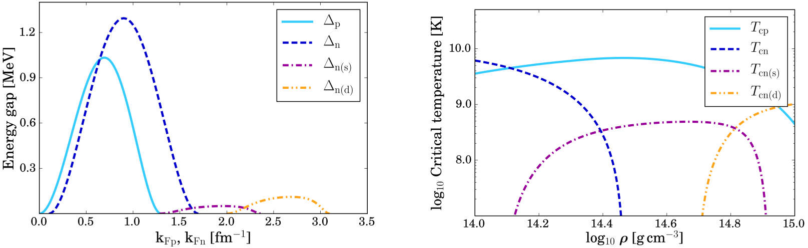


Figure 1: Left: Parametrised proton (singlet) and neutron (singlet, triplet) energy gaps as a function of Fermi wave numbers (Ho, Glampedakis & Andersson, 2012). Right: Critical temperatures of superconductivity/superfluidity as a function of the neutron star density. The values are computed for the NRAPR equation of state (Steiner et al., 2005; Chamel, 2008).

The (old) type-II picture

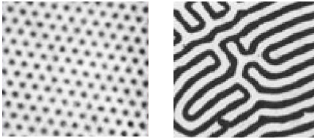


Figure 2: Type-II and intermediate type-I state (Brandt & Essmann, 1987).

- Due to high conductivity, the magnetic flux cannot be expelled from their interiors $\Rightarrow$ neutron stars do not exhibit Meissner effect and are in a **metastable** state (Baym, Pethick & Pines, 1969; Ho, Andersson & Graber, 2017).

- State depends on **characteristic lengthscales** and standard considerations give $\kappa = \lambda_\star/\xi_{\rm ft} > 1/\sqrt{2}$ in the outer core, i.e., a **type-II state** with

$$H_{c1} = 4\pi\mathcal{E}_{\rm ft}/\phi_0 \sim 10^{14}\,\mathrm{G}, \qquad H_{c2} = \phi_0/(2\pi\xi_{\rm ft}^2) \sim 10^{15}\,\mathrm{G}. \tag{1}$$

- Each fluxtube carries a **flux quantum** $\phi_0 = hc/2e \approx 2.1 \times 10^{-7}\,\mathrm{G\,cm^2}$. All flux quanta add up to the total magnetic flux, so that the **averaged magnetic induction** is related to the fluxtube area density $\mathcal{N}_{\rm ft}$:

$$B = \mathcal{N}_{\rm ft}\phi_0, \quad \rightarrow \quad \mathcal{N}_{\rm ft} \approx 4.8 \times 10^{18}\,\left(B/10^{12}\,\mathrm{G}\right)\mathrm{cm}^{-2}. \tag{2}$$

Motivation

- Our understanding of **macroscopic NS superconductivity** is mainly based on **time-independent equilibrium** and **single-component** considerations.
- It is unclear what happens in the **stellar interior at different densities** as the star cools below T_c and when entrainment is included $\Rightarrow$ how can we better understand the **SC phase** and its formation?
- Focus on the **two-condensate aspect** and expand on **earlier works** by Alpar, Langer & Sauls (1984); Charbonneau & Zhitnitsky (2007); Alford & Good (2008); Kobyakov et al. (2015); Haber & Schmitt (2017).

Rely on techniques for laboratory systems to construct **phase diagrams** of the protons by deducing their ground state in the presence of a magnetic field as a function of density.

Galilean invariant GL model

- With entrainment, **velocity-dependent terms** in energy density read

$$F_{\text{vel}} = \tfrac{1}{2}\rho_{\mathrm{p}}|\mathbf{V}_{\mathrm{p}}|^2 + \tfrac{1}{2}\rho_{\mathrm{n}}|\mathbf{V}_{\mathrm{n}}|^2 - \tfrac{1}{2}\rho^{\mathrm{pn}}|\mathbf{V}_{\mathrm{p}} - \mathbf{V}_{\mathrm{n}}|^2\,, \tag{3}$$

where ρ_{p} and ρ_{n} are the true mass densities, the coefficient $\rho^{\mathrm{pn}} < 0$ determines the strength of entrainment (Andreev & Bashkin, 1975) and $\mathbf{V}_{\mathrm{p,n}}$ are superfluid velocities related to canonical momenta, i.e., $\propto \nabla \arg\psi_{\mathrm{x}}$.

- In a mean-field framework, entrainment first enters at 4th order in $\psi_{\mathrm{n,p}}$ and 2nd order in their derivatives, i.e., we require a linear combination of

$$|\psi_x|^2|\boldsymbol{\nabla}\psi_y|^2,\ \psi_x\psi_y\boldsymbol{\nabla}\psi_x^\star\cdot\boldsymbol{\nabla}\psi_y^\star,\ \psi_x\psi_y^\star\boldsymbol{\nabla}\psi_x^\star\cdot\boldsymbol{\nabla}\psi_y,\ \psi_x^\star\psi_y^\star\boldsymbol{\nabla}\psi_x\cdot\boldsymbol{\nabla}\psi_y, \tag{4}$$

where $x, y \in \{\mathrm{p}, \mathrm{n}\}$. Galilean invariance can be used to simplify the sum and is crucial to link our energy density to **nuclear physics models**.

- Total Helmholtz free energy density is obtained by adding entrainment terms to the usual free energy of a two-component superconductor, and introducing the magnetic vector potential **A** by **minimal coupling**:

$$\begin{aligned} F[\psi_{\mathrm{p}},\psi_{\mathrm{n}},\mathbf{A}] = F_0 &- \mu_{\mathrm{p}}|\psi_{\mathrm{p}}|^2 - \mu_{\mathrm{n}}|\psi_{\mathrm{n}}|^2 + \frac{g_{\mathrm{pp}}}{2}|\psi_{\mathrm{p}}|^4 + \frac{g_{\mathrm{nn}}}{2}|\psi_{\mathrm{n}}|^4 + g_{\mathrm{pn}}|\psi_{\mathrm{p}}|^2|\psi_{\mathrm{n}}|^2 \\ &+ \frac{\hbar^2}{4m_{\mathrm{u}}}\left|\left(\boldsymbol{\nabla} - \frac{2ie}{\hbar c}\mathbf{A}\right)\psi_{\mathrm{p}}\right|^2 + \frac{\hbar^2}{4m_{\mathrm{u}}}|\boldsymbol{\nabla}\psi_{\mathrm{n}}|^2 + \frac{1}{8\pi}|\boldsymbol{\nabla}\times\mathbf{A}|^2 \\ &+ h_1\left|\left(\boldsymbol{\nabla} - \frac{2ie}{\hbar c}\mathbf{A}\right)(\psi_{\mathrm{n}}^{\star}\psi_{\mathrm{p}})\right|^2 + \tfrac{1}{2}(h_2 - h_1)\boldsymbol{\nabla}(|\psi_{\mathrm{p}}|^2)\cdot\boldsymbol{\nabla}(|\psi_{\mathrm{n}}|^2) \\ &+ \tfrac{1}{4}h_3\left(\left|\boldsymbol{\nabla}(|\psi_{\mathrm{p}}|^2)\right|^2 + \left|\boldsymbol{\nabla}(|\psi_{\mathrm{n}}|^2)\right|^2\right), \end{aligned} \tag{5}$$

where F_0 is an arbitrary reference level and proton Cooper pairs have charge $2\mathrm{e}$. μ_{p} and μ_{n} are the chemical potentials, while g_{pp} and g_{nn} define the self-repulsion of the condensates, and g_{pn} their mutual repulsion.

- We connect this functional to the **Skyrme interaction** to obtain coefficients h_i that allow a **realistic description** of the neutron star interior:

$$h_1 = C_0^{\tau} - C_1^{\tau}\,, \qquad h_2 = -4C_0^{\Delta\rho} + 4C_1^{\Delta\rho}\,, \tag{6}$$

$$h_3 = h_4 = C_0^{\tau} + C_1^{\tau} - 4C_0^{\Delta\rho} - 4C_1^{\Delta\rho}\,. \tag{7}$$

- The parameter h_1 controls the entrainment (Chamel & Haensel, 2006)

$$\rho^{\rm pn} = -\frac{2}{\hbar^2} h_1 \rho_{\rm n}\rho_{\rm p}. \tag{8}$$

- We also use the Skyrme model to determine the **stellar composition** (solving for baryon conservation, charge neutrality, beta equilibrium and muon production rate) (Chamel, 2008), but a separate **parametrisation** for the **SF/SC gaps** (Andersson, Comer & Glampedakis, 2005; Ho et al., 2015).

Two thought experiments

- To find the **ground state** in the presence of an **imposed magnetic field**, we can control (i) the magnetic flux density, $\mathbf{B} = \nabla \times \mathbf{A}$, by imposing a mean flux $\overline{B}$, or (ii) the thermodynamic external magnetic field $\mathbf{H}$.

- Case (i) approximates the **neutron star interior**, which becomes superconducting as the star cools in the presence of a pre-existing field.

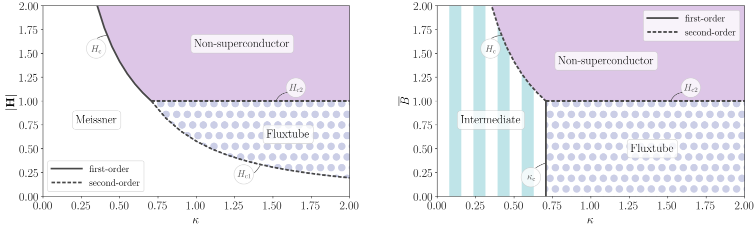
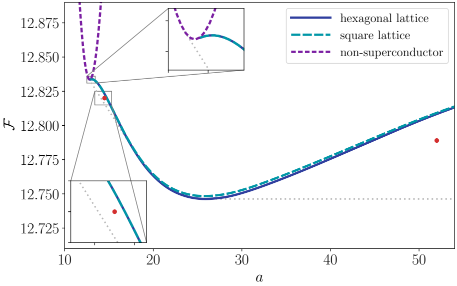


Figure 3: Phase diagrams for a one-component superconductor, for different values of the Ginzburg-Landau parameter, κ. The experiment with an imposed external field, $|\mathbf{H}|$, in nondimensional units is shown on the left, while the right panel shows the phase transitions in the experiment with an imposed mean flux, $\overline{B}$.

Energy per flux quantum

- We solve the Euler-Lagrange equations with **quasi-periodic boundary conditions** (Wood et al., 2019), which involves specifying the domain size $L_x \times L_y$, and the number N of magnetic flux quanta within the domain $\Rightarrow$ different choices allow comparing **square** and **hexagonal lattices**.

- The Helmholtz free energy per magnetic flux quantum per unit length is



$$\mathcal{F} \equiv \frac{1}{N} \int_{x=0}^{L_x} \int_{y=0}^{L_y} F \,\mathrm{d}x\,\mathrm{d}y\,. \quad (9)$$

Figure 4: Helmholtz free energy per flux quantum per unit length, $\mathcal{F}$, as a function of the area per magnetic flux quantum, $a = 2\pi/\overline{B}$, for the NRAPR EoS at $n_\mathrm{b} = 0.2831/\mathrm{fm}^3$. The energy in the square (long-dashed, cyan) and hexagonal (solid, blue) lattice states matches smoothly onto the energy of the non-superconducting state (short-dashed, purple) at $a \simeq 12.9$.

SC phases — Inhomogeneous ground states

- Choosing a sufficiently large domain, and values of a, we obtain examples of **inhomogeneous ground states** $\Rightarrow$ for NRAPR at $n_{\mathrm{b}} = 0.2831/\mathrm{fm}^3$ with $a = 14.5$ plus $N = 24$ (left) and $a = 52$ plus $N = 14$ (right).
- In both cases, the aspect ratio is $\sqrt{3}$ and the **pure hexagonal lattice** a possible state but not the ground state $\Rightarrow \mathcal{F}$ is lower than for pure lattice.

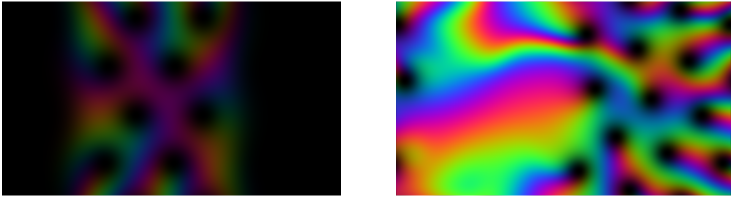


Figure 5: Inhomogeneous ground states for NRAPR. Brightness and hue indicate density and phase of the proton order parameter, ψ_{p}, respectively. The left panel shows a mixture of non-superconducting protons and hexagonal fluxtube lattice, while the right one is mixture of Meissner state and hexagonal fluxtube lattice.

Phase diagrams LNS

- When such mixed states are present, second-order phase transitions are replaced by **first-order transitions** at $H_{c1'} < H_{c1}$ and $H_{c2'} > H_{c2}$.
- We can determine **critical fields** (partially semi-analytically, partially numerically) and construct phase-diagrams of the superconducting state throughout the neutron star core. For the **LNS** equation of state:

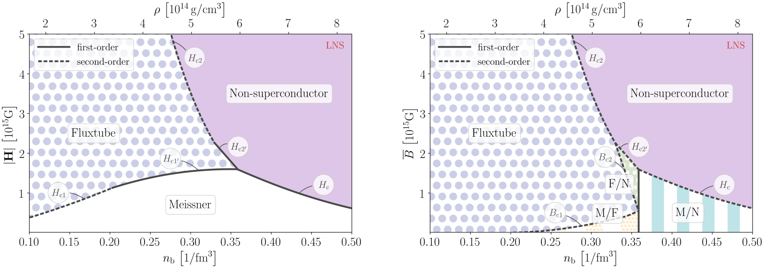
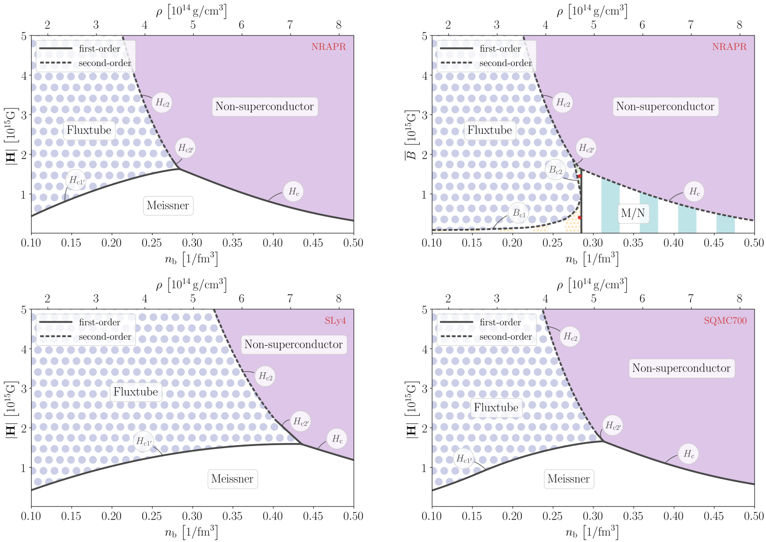


Figure 6: Phase diagrams for LNS. There are inhomogeneous regimes of the Meissner/Non-superconducting (M/N), Meissner/Fluxtube (M/F) and Fluxtube/Non-superconducting (F/N) states.

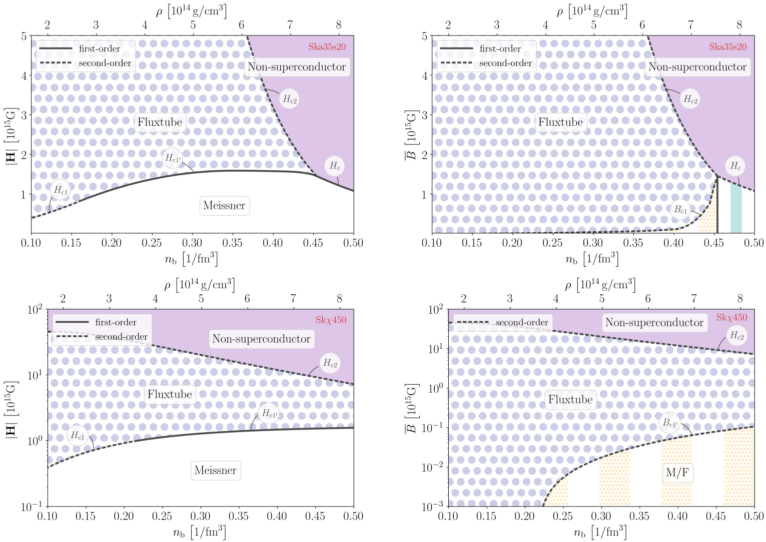
SC phases

NRAPR, SLy4, SQMC700



SC phases

Ska35s20, Skχ450



SC phases

Implications

- Entrainment causes **type-1.5 SC** due to fluxtube repulsion on short scales and attraction on large scales $\Rightarrow$ when imposing $\overline{B}$, mixed states appear.

- For typical EoSs, the outer core of pulsars with $\lesssim 10^{14}$ G is not a type-II superconductor but mainly a type-1.5 system, where **magnetic flux is irregularly distributed** and **retained**.

- In the inner core, flux is distributed in an **intermediate type-I state**. The transition can be estimated via:

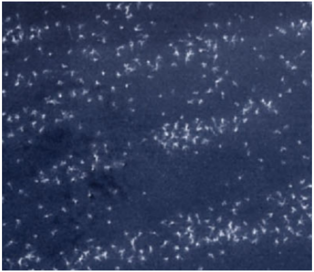
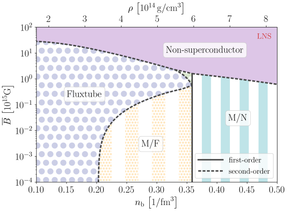


Figure 7: Magnetic decoration image of multi-band superconductor Mg2B (Moshchalkov et al., 2009).

$$\boxed{\frac{\lambda_\star}{\xi_{\mathrm{ft}}} = \frac{1}{\sqrt{2}}\sqrt{1 + h_1\,\frac{n_{\mathrm{n}}}{n_{\mathrm{p}}}}.} \qquad (10)$$

SC phases

Flux distribution



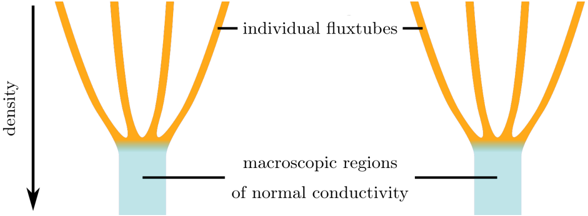


Figure 8: Zoomed-out version of the LNS phase diagram for a fixed mean flux (left). Schematic representation of the magnetic flux at the transition from outer to inner core (right).

- At low densities, protons are in a **type-1.5 regime**, where flux is quantized into thin fluxtubes (orange) of preferred separation (overall confined to a small fraction of the total volume).

- In the **type-I inner core**, flux is contained in macroscopic regions of normal conductivity (light blue) that alternate with flux-free regions.

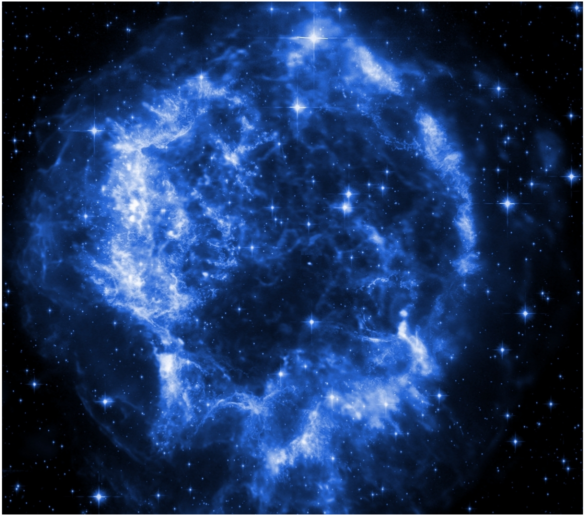
(A few) caveats

- We assume that the **Skyrme model** correctly describes interactions up to the neutron star centre. If exotic particles / non-nucleonic matter are present, this would modify the picture at high densities.

- Our Ginzburg–Landau model is **time-independent** and neglects **rotation**, i.e., we do not capture dynamics or incorporate neutron vortices, which are crucial to get the full macroscopic picture.

- For a full dynamical model, we would need to incorporate the **electron component**. However, they are **normal** and do not form a quantum condensate. We do not have a formalism to consistently include such a (particle) component in the Ginzburg–Landau model.

The end



Thought experiments

- To find the **ground state** for our system in the presence of an **imposed magnetic field**, we can perform two distinct experiments: we control (i) the magnetic flux density, $\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$, by imposing a mean or net magnetic flux, or (ii) the thermodynamic external magnetic field, $\mathbf{H}$.

- In the first case, we minimise the **Helmholtz free energy**, $\mathcal{F} = \langle F \rangle$, where the angled brackets represent some kind of integral over our physical domain $\Rightarrow$ closely approximates the conditions in the neutron star core, which becomes superconducting as the star cools in the presence of pre-existing magnetic field. The ground state can be **inhomogeneous**.

- In the second case, we minimise the dimensionless **Gibbs free energy**, $\mathcal{G} = \mathcal{F} - 2\kappa^2 \mathbf{H} \cdot \langle \mathbf{B} \rangle$. In an unbounded domain, the ground state is guaranteed to be **homogeneous**, and the phase diagram simpler.

Euler-Lagrange equations

- Whether we work with $\mathcal{F}$ or $\mathcal{G}$, we obtain the same equations of motion:

$$\kappa^2 \boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{A}) = \Im \left\{ \psi_{\rm p}^\star (\boldsymbol{\nabla} - i\mathbf{A}) \psi_{\rm p} + \frac{h_1}{\epsilon} \psi_{\rm n} \psi_{\rm p}^\star (\boldsymbol{\nabla} - i\mathbf{A})(\psi_{\rm n}^\star \psi_{\rm p}) \right\} , \qquad (11)$$

$$\begin{aligned} \nabla^2 \psi_{\rm n} &= R^2 (|\psi_{\rm n}|^2 - 1)\psi_{\rm n} + \alpha(|\psi_{\rm p}|^2 - 1)\psi_{\rm n} \\ &\quad - h_1 \psi_{\rm p} (\boldsymbol{\nabla} + i\mathbf{A})^2 (\psi_{\rm p}^\star \psi_{\rm n}) \\ &\quad - \psi_{\rm n} \nabla^2 \left(\frac{h_2 - h_1}{2} |\psi_{\rm p}|^2 + \frac{h_3}{2\epsilon} |\psi_{\rm n}|^2 \right) , \end{aligned} \qquad (12)$$

$$\begin{aligned} (\boldsymbol{\nabla} - i\mathbf{A})^2 \psi_{\rm p} &= (|\psi_{\rm p}|^2 - 1)\psi_{\rm p} + \frac{\alpha}{\epsilon}(|\psi_{\rm n}|^2 - 1)\psi_{\rm p} \\ &\quad - \frac{h_1}{\epsilon} \psi_{\rm n} (\boldsymbol{\nabla} - i\mathbf{A})^2 (\psi_{\rm n}^\star \psi_{\rm p}) \\ &\quad - \psi_{\rm p} \nabla^2 \left(\frac{h_2 - h_1}{2\epsilon} |\psi_{\rm n}|^2 + \frac{h_3}{2} |\psi_{\rm p}|^2 \right) . \end{aligned} \qquad (13)$$

Energy minimisation

- F is approximated numerically on a regular 2D grid. The order parameters $\psi_{\rm p}$ and $\psi_{\rm n}$ are defined on the gridpoints as $\psi_{\rm p}^{i,j}$ and $\psi_{\rm n}^{i,j}$, while the vector field $\mathbf{A}$ has two components, (A_x, A_y), defined on the corresponding links between the gridpoints, i.e., we have $A_x^{i+1/2,j}$ and $A_y^{i,j+1/2}$.

- The gauge coupling between $\psi_{\rm p}$ and $\mathbf{A}$ is implemented using a **Peierls substitution** to preserve (discrete) gauge symmetry, e.g.,

$$\left|\left(\frac{\partial}{\partial x} - \mathrm{i}A_x\right)\psi_{\rm p}\right| = \left|\frac{\partial}{\partial x}\exp(-\textstyle\int \mathrm{i}A_x\,\mathrm{d}x)\psi_{\rm p}\right|$$
$$\Rightarrow \left|\left(\frac{\partial}{\partial x} - \mathrm{i}A_x\right)\psi_{\rm p}\right|^{i+1/2,j} \simeq \frac{1}{\delta x}\left|\exp(-\mathrm{i}A_x^{i+1/2,j}\,\delta x)\psi_{\rm p}^{i+1,j} - \psi_{\rm p}^{i,j}\right|. \qquad (14)$$

- This leads to a discrete approximation $\mathcal{F}_{\rm dis}[\psi_{\rm p}^{i,j}, \psi_{\rm n}^{i,j}, A_x^{i+1/2,j}, A_y^{i,j+1/2}]$ and we obtain the ground state using a gradient-descent, iteration method.

SC formation

- In the outer core, initially, only protons are superconducting (neutrons remain normal), so we model the **formation of the superconducting phase** with a single-component time-dependent Ginzburg-Landau model.

- Study the **dynamics of the phase transition** under different circumstances in analogy to numerical experiments of laboratory systems (Liu, Mondello & Goldenfeld, 1991; Frahm, Ullah & Dorsey, 1991).

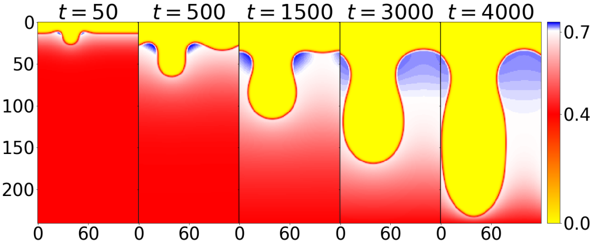


Figure 9: Evolution of the magnetic field in a type-I system in the nucleation regime.

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