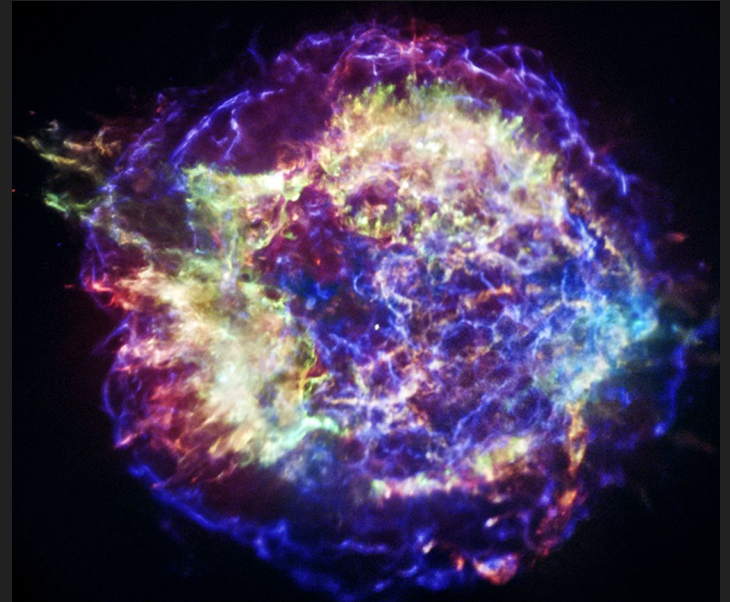


Superfluid Spin-up: 3D Simulations of Post-Glitch Dynamics in Neutron Star Cores

[arXiv:2407.18810](https://arxiv.org/abs/2407.18810)

Dr Vanessa Graber
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in collaboration with J. Rafael Fuentes
(University of Colorado Boulder)



Cassiopeia A supernova remnant
(credit: NASA/CXC/SAO)

Neutron star interiors

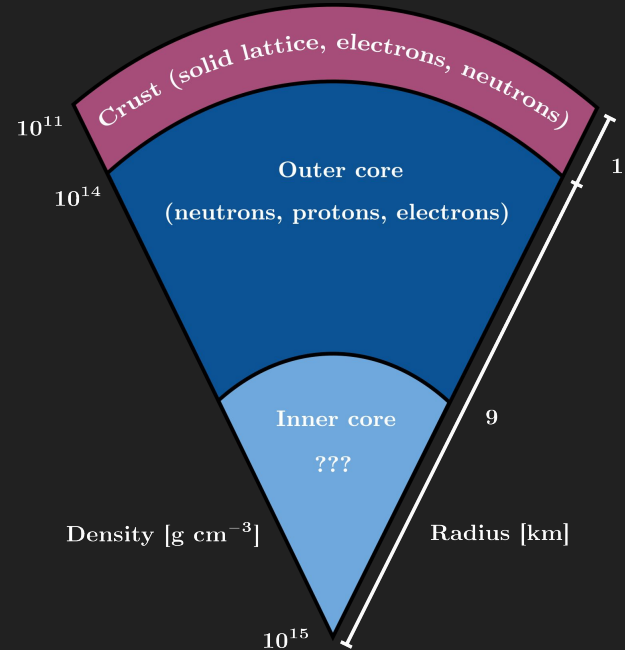
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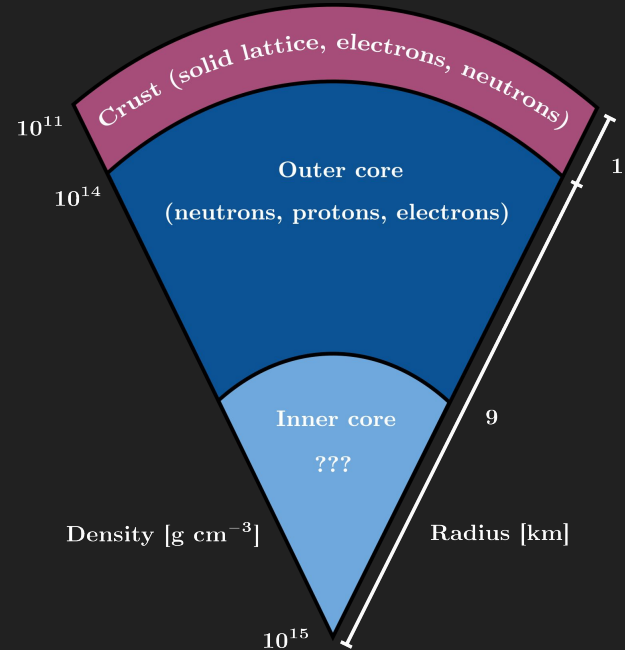
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We can decompose neutron stars into solid crusts and fluid cores.



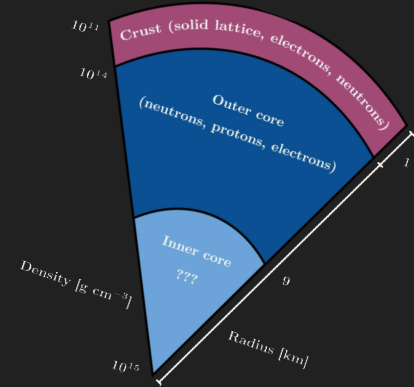
Superfluid components

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Interiors are well below neutron and proton Fermi temperatures ($\sim 10^{12}$ K).

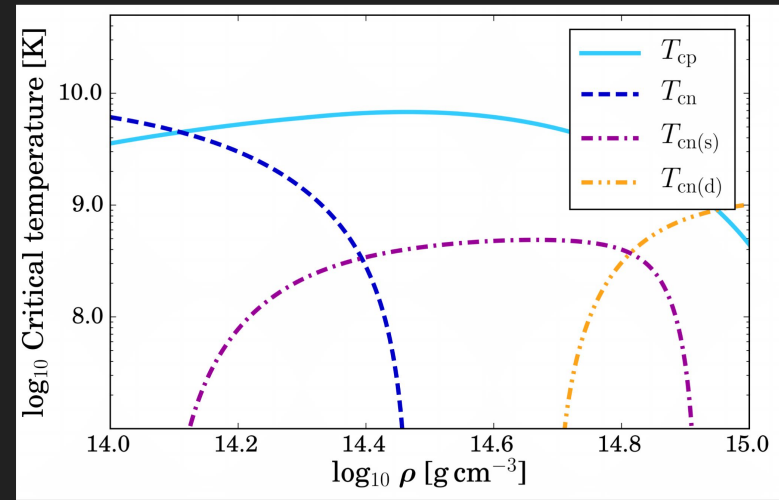
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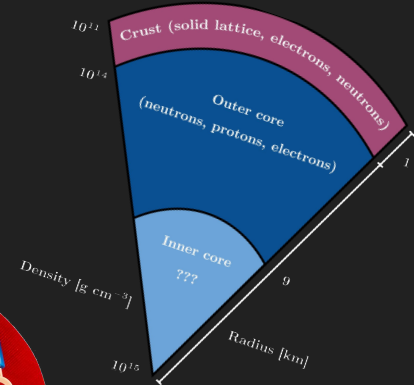
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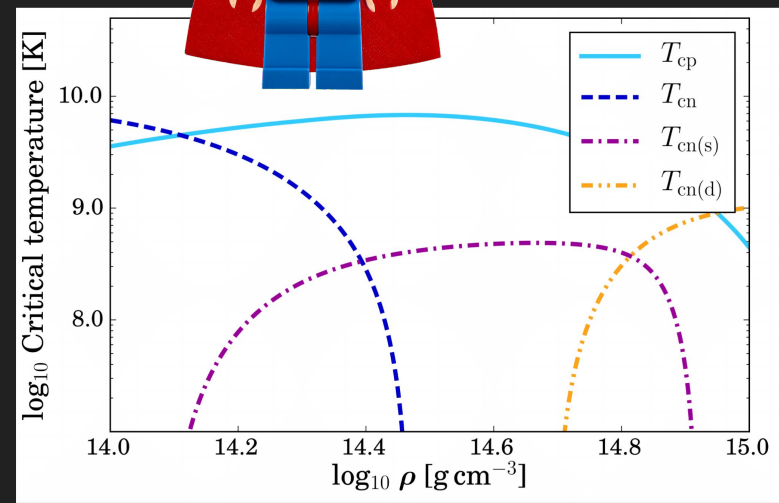
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Neutron stars contain at least 3 superfluid components.



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Quantum vortices



Credit: NOAA Photo Library

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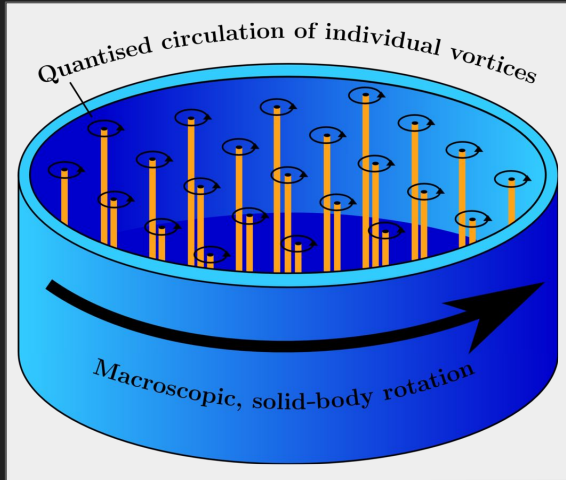
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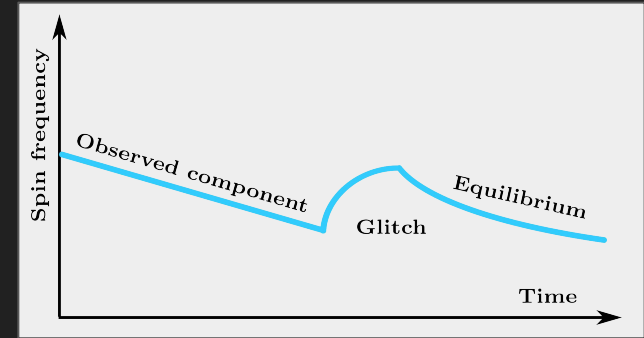
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- The vortices form an array that mimics solid-body rotation on large scales $\boldsymbol{\omega} = 2\boldsymbol{\Omega} = N_v \boldsymbol{\kappa}$.

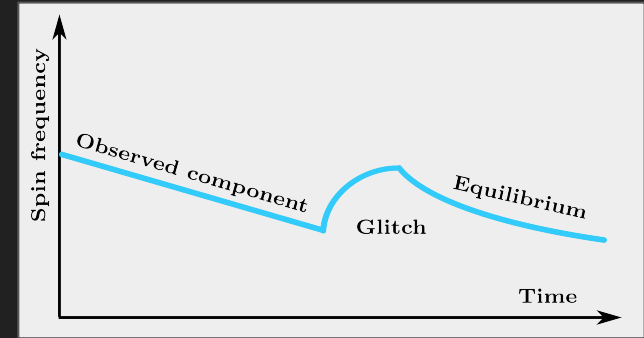
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- As observed by pulsar timing, the regular spin-down of neutron stars can be interrupted by sudden spin-ups.



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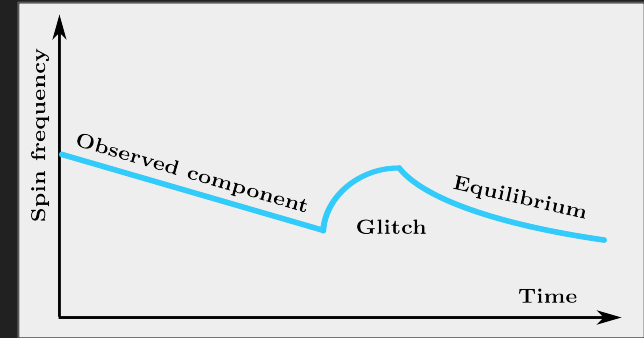
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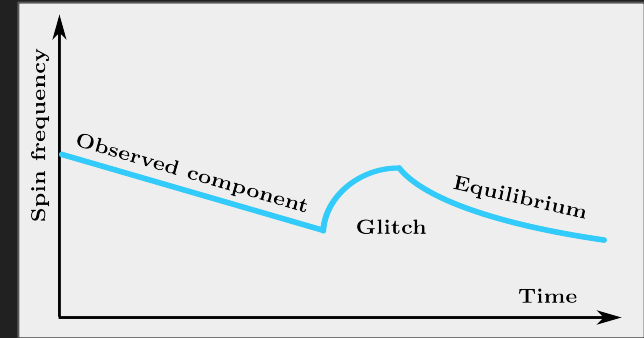


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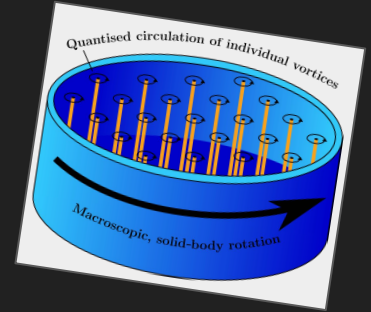
cooked

raw

Polar glitches II

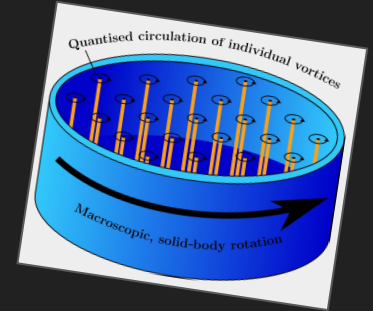
- Spin-up glitches can be naturally explained in a multi-component neutron star model.

Superfluid spin-down can be impeded by pinning of vortices to crustal lattice.



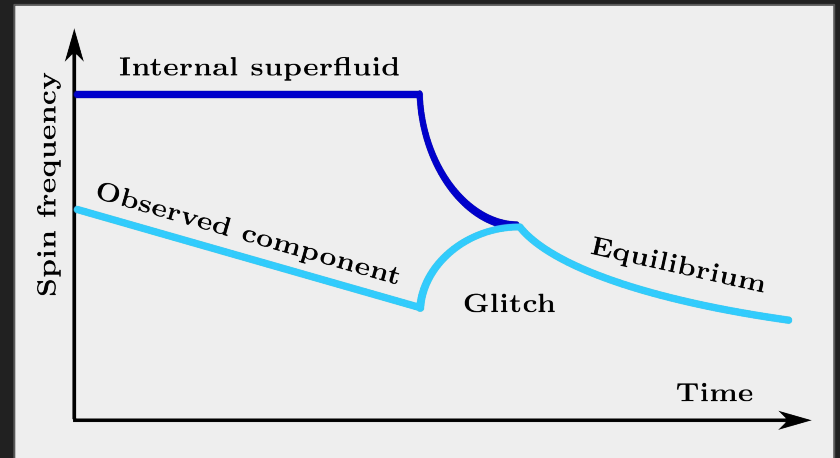
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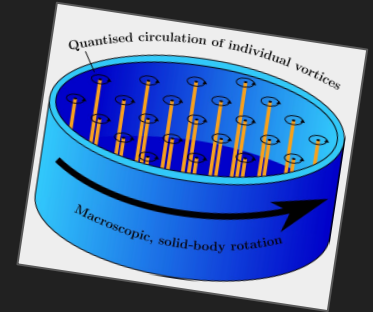
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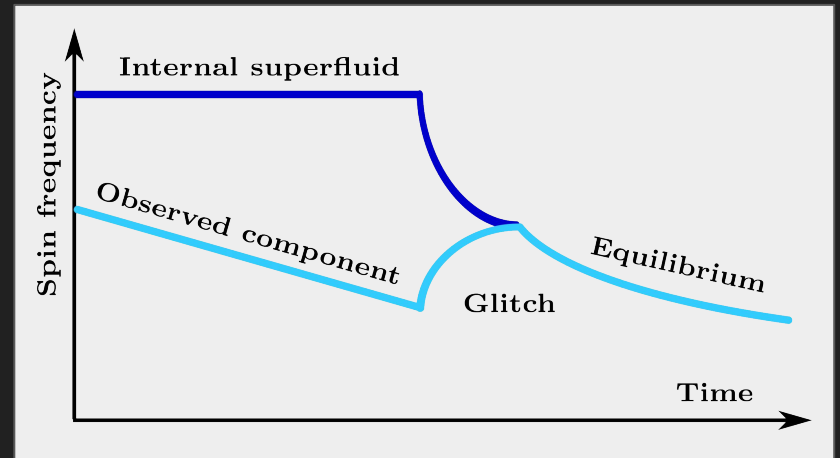
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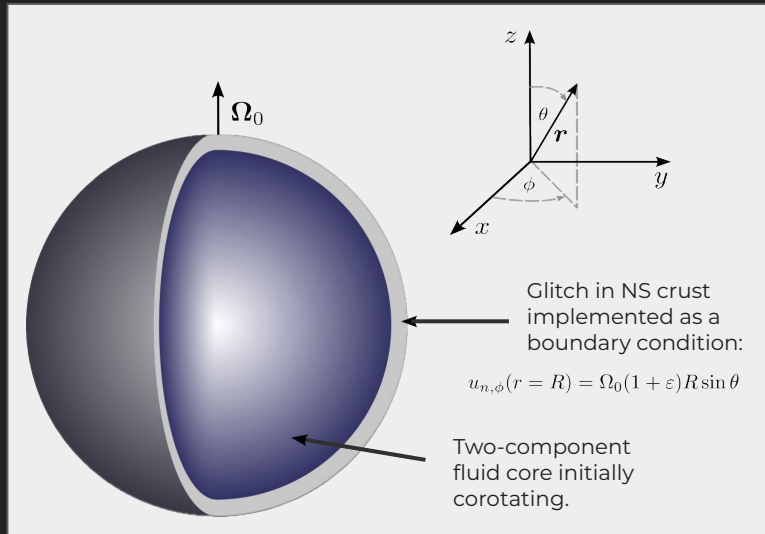
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The shape of the glitch encodes the (hidden) internal neutron star physics.



Numerical experiment set-up

- In our new study, we focus on the response of a two-component fluid core to a glitch that is driven by the crustal superfluid.

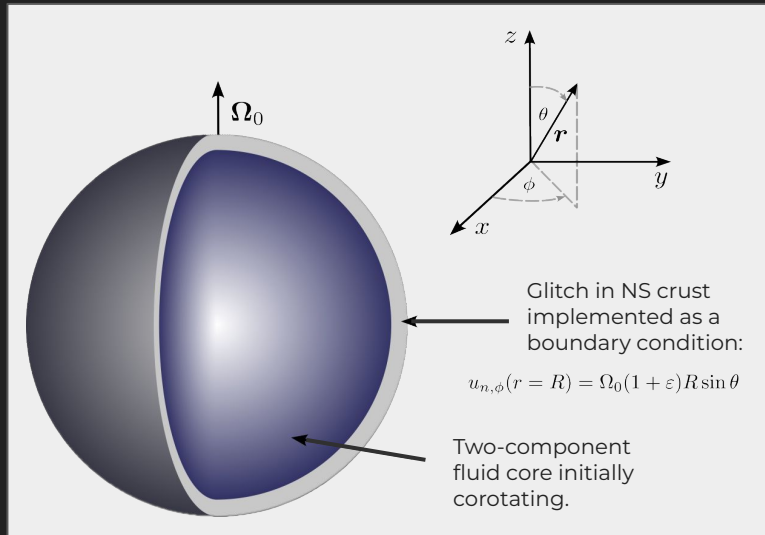


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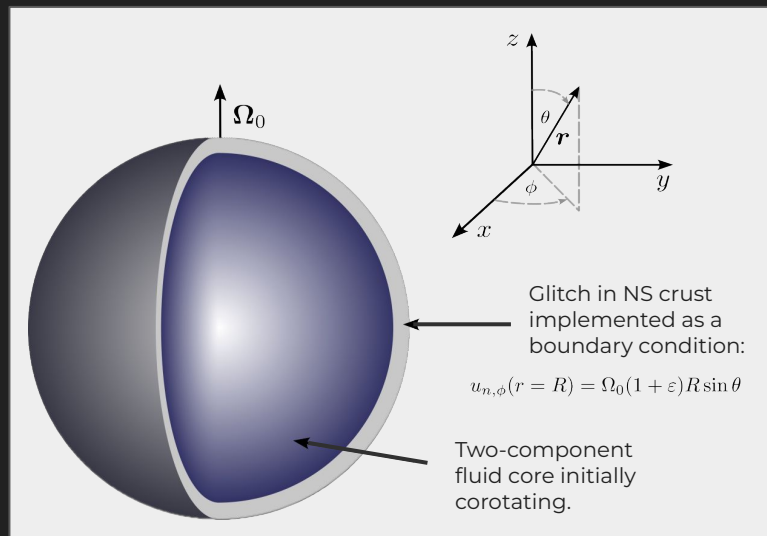
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Goal: Study the core's response to the glitch in full sphere for the first time.

HVBK Equations

- We focus on the hydrodynamical picture and solve the Hall–Vinen–Bekarevich–Khalatnikov (HVBK) equations initially developed for laboratory superfluid helium with the pseudo-spectral code Dedalus:

$$\frac{\partial \mathbf{u}_n}{\partial t} + \mathbf{u}_n \cdot \nabla \mathbf{u}_n = -\nabla \tilde{\mu}_n + \nu \nabla^2 \mathbf{u}_n + \frac{\mathbf{F}_{\text{MF}}}{\rho_n}, \quad (1)$$

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$$\mathbf{F}_{\text{MF}} = \rho_s [\mathcal{B} (\hat{\omega}_s \times (\omega_s \times \mathbf{u}_{sn})) + \mathcal{B}' (\omega_s \times \mathbf{u}_{sn})], \quad (4)$$

with $\mathbf{u}_{sn} = \mathbf{u}_s - \mathbf{u}_n, \omega_s = \nabla \times \mathbf{u}_s$

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\mathbf{F}_{MF} is the mutual friction force due to interactions of vortices and their surroundings.

Characteristic time scales

- For realistic neutron stars, we estimate the mutual friction and Ekman timescale as follows:

$$\tau_{\text{MF}} \sim \frac{1}{2\Omega_s \mathcal{B}} \sim 80 \text{ s} \left(\frac{P_{\text{rot}}}{0.1 \text{ s}} \right) \left(\frac{\mathcal{B}}{10^{-4}} \right)^{-1}$$

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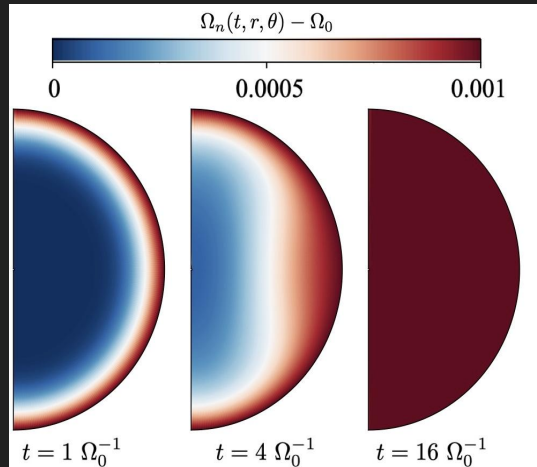
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Note: Due to numerical constraints, we vary $B \sim 1-10^{-3}$ with $B/B' \sim 2$ and set $\text{Ek} \sim 5 \times 10^{-3(4)}$.

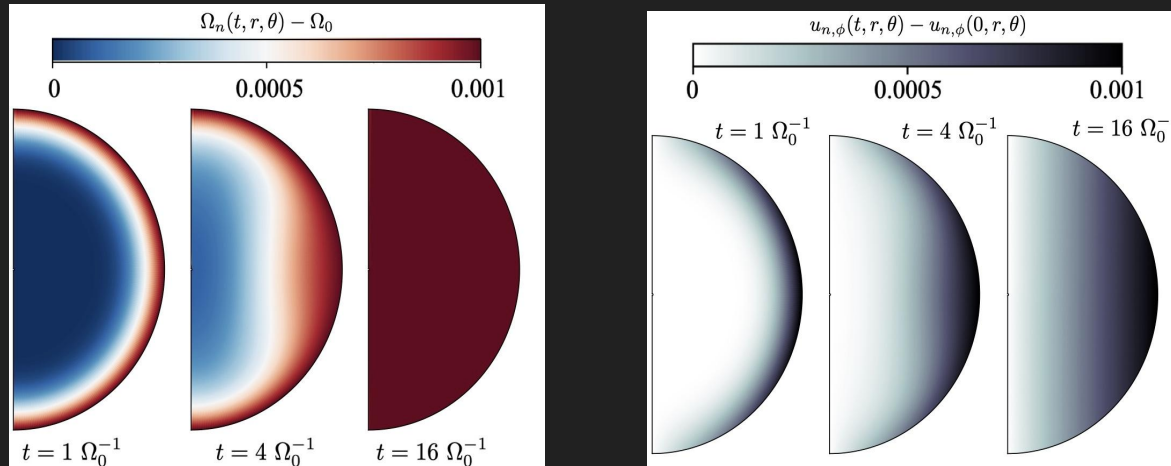
Spinning up a single-component fluid I

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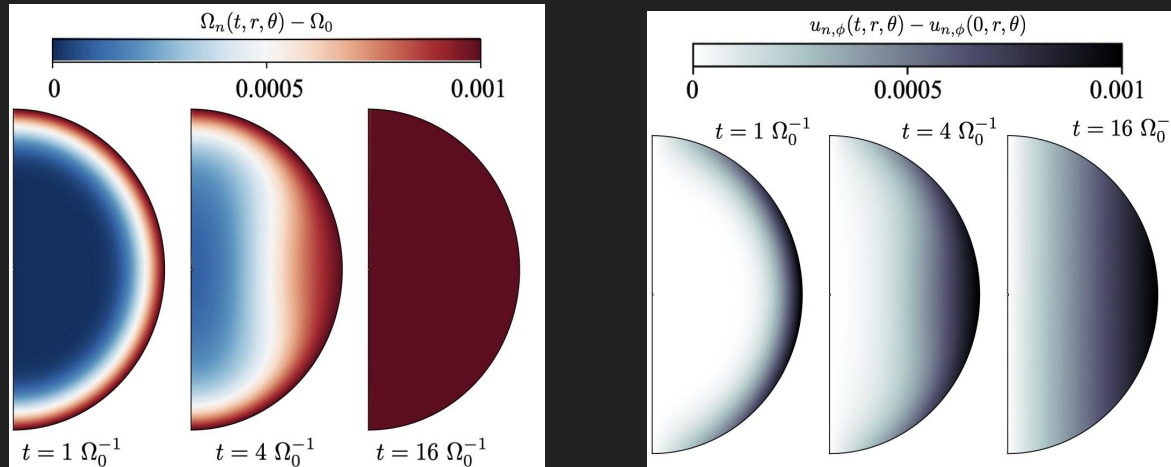
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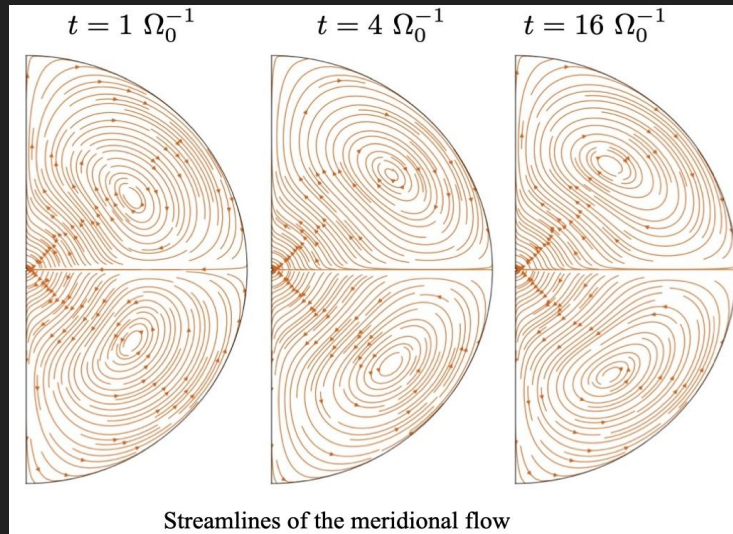


The axisymmetric behaviour is a direct result of the Taylor-Proudman theorem.

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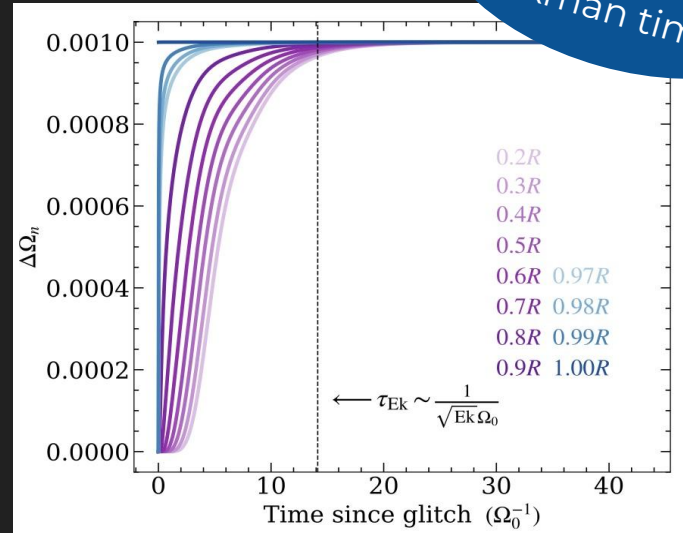
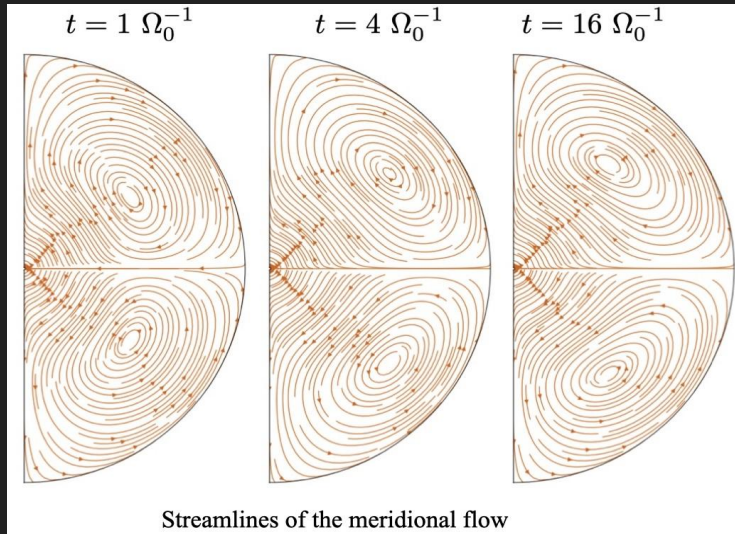
- Ekman pumping leads to the formation of a stable circular flow pattern in each semi-hemisphere.



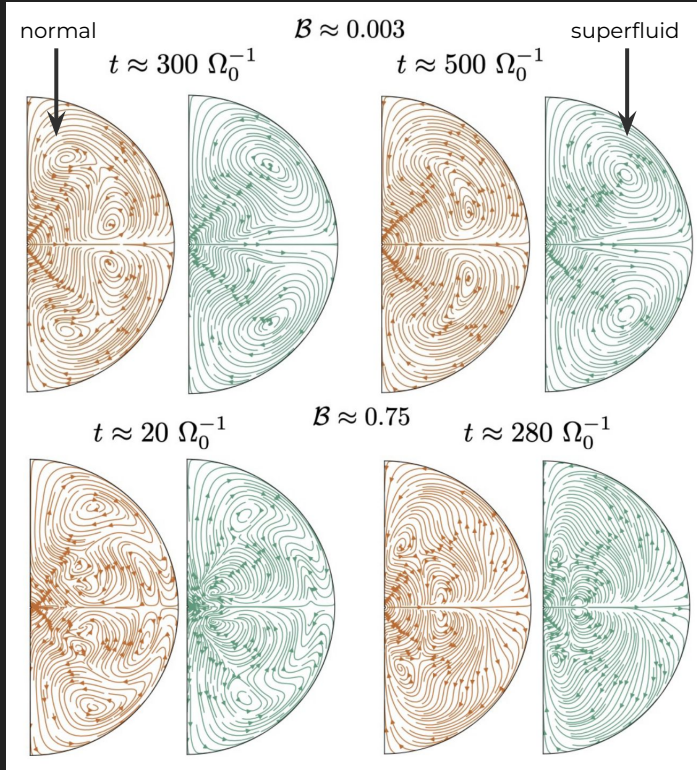
Spinning up a single-component fluid II

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As expected, a viscous fluid spins up on the Ekman time.

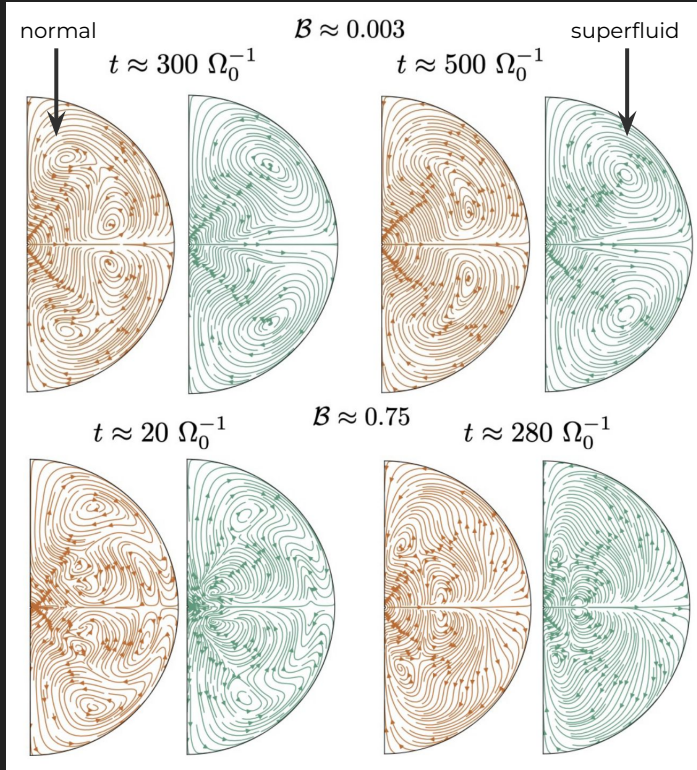


Spinning up a two-component fluid I



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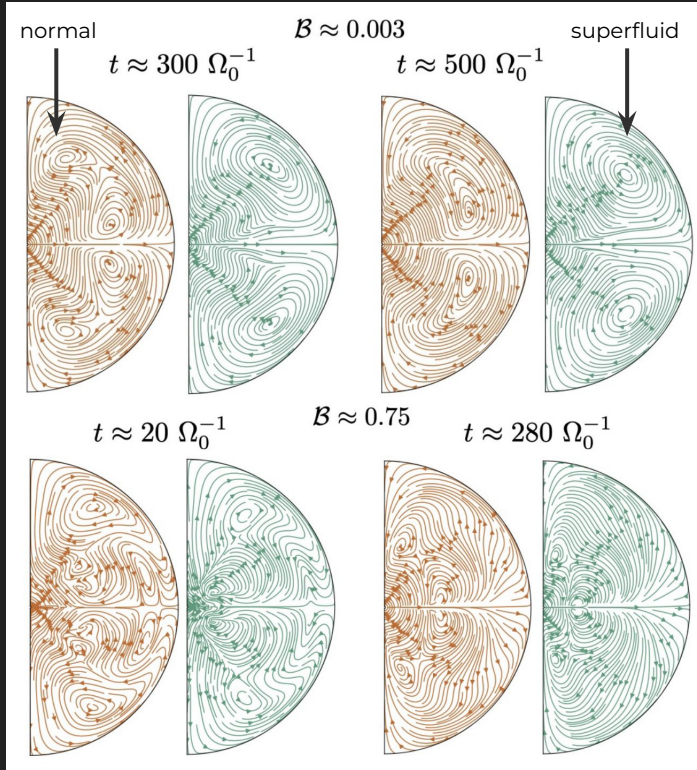


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For strong coupling, the superfluid follows the viscous fluid pattern.

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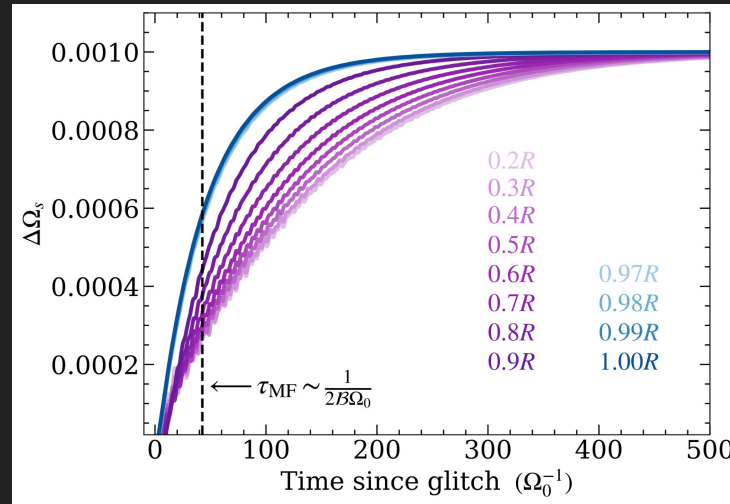
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For strong coupling, the superfluid follows the viscous fluid pattern.

For $B \sim 0.75$, the normal fluid no longer develops the single cell flow structure.

Spinning up a two-component fluid II

- While the initial evolution is qualitatively similar and we obtain constant azimuthal velocities over cylindrical surfaces, the spin-up of the superfluid is delayed because of the mutual friction coupling.

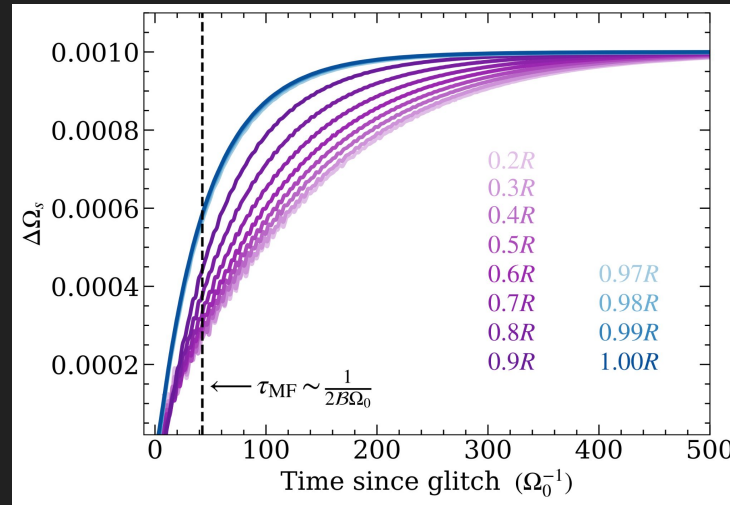


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Superfluid can only accelerate once the Ekman pumping has spun up the viscous component.

Once differential rotation builds up, the superfluid is coupled via mutual friction.

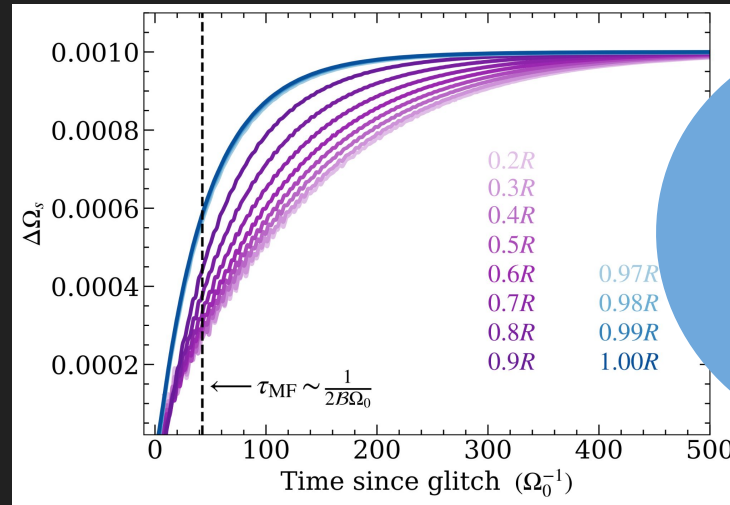


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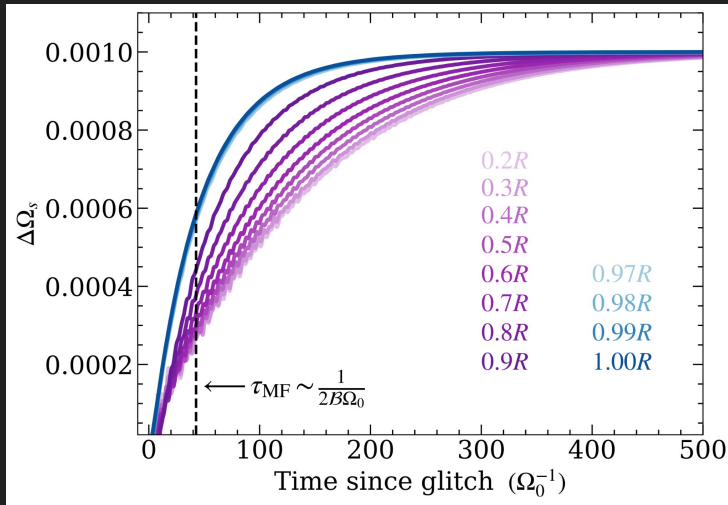


Outer superfluid layers spin up on the mutual friction timescale, while it takes longer for the inner layers.

Spinning up a two-component fluid III

- To extract the spin-up timescale, we fit

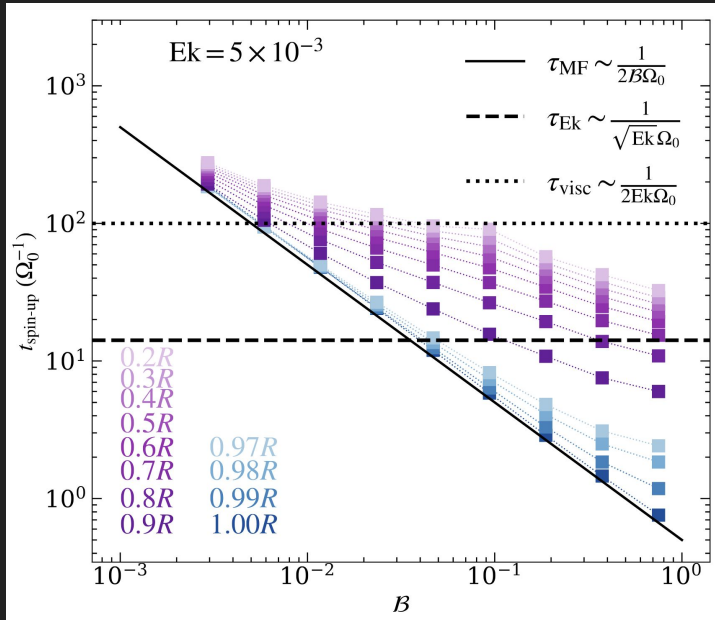
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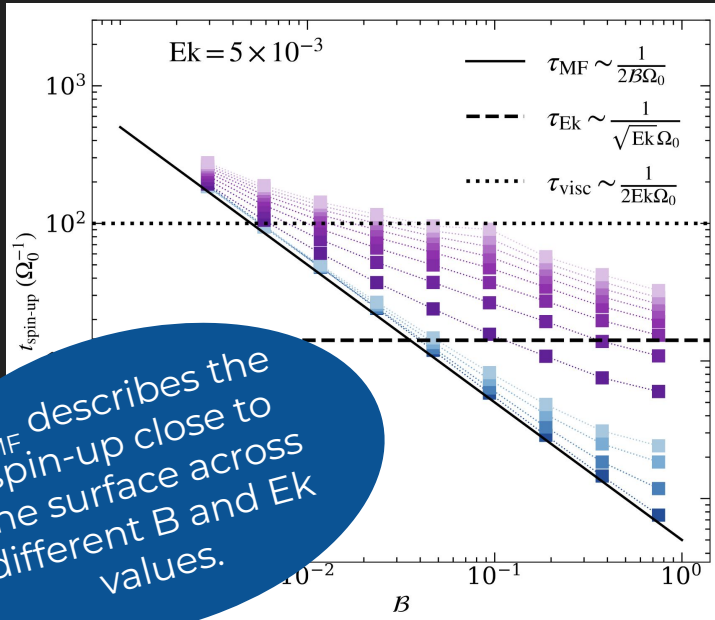
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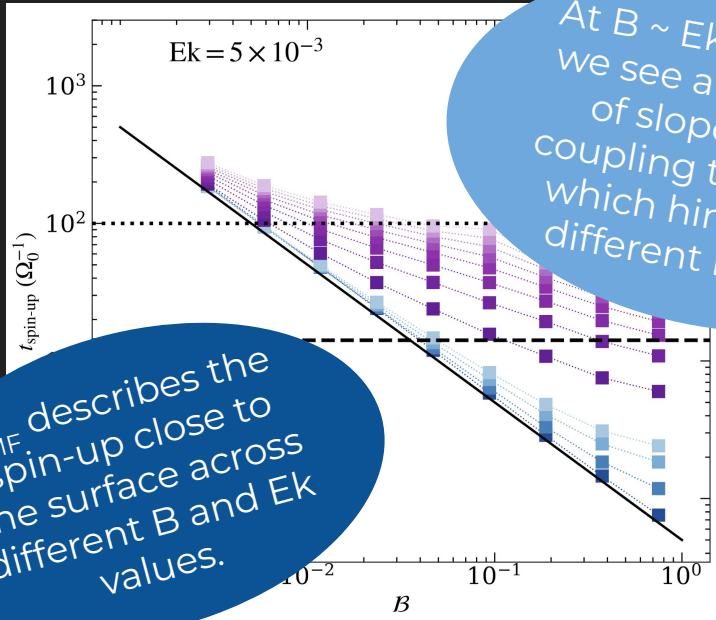


τ_{MF} describes the spin-up close to the surface across different B and Ek values.

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At $B \sim Ek^{0.5}$ ($B \sim 0.07$), we see a separation of slopes in our coupling timescales, which hints at two different regimes.

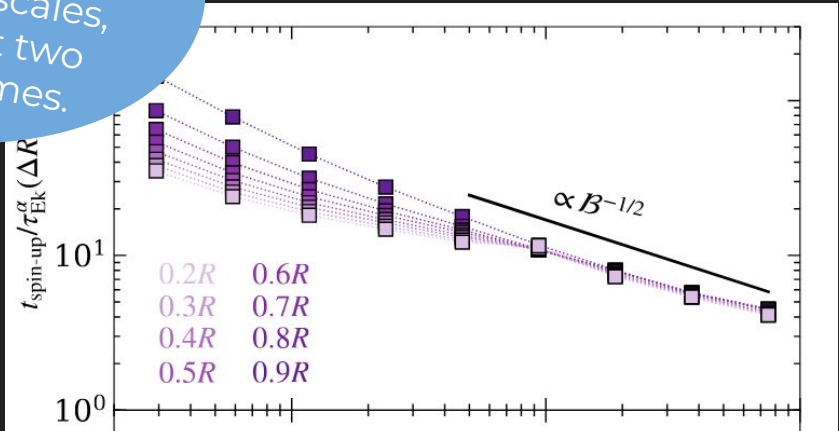
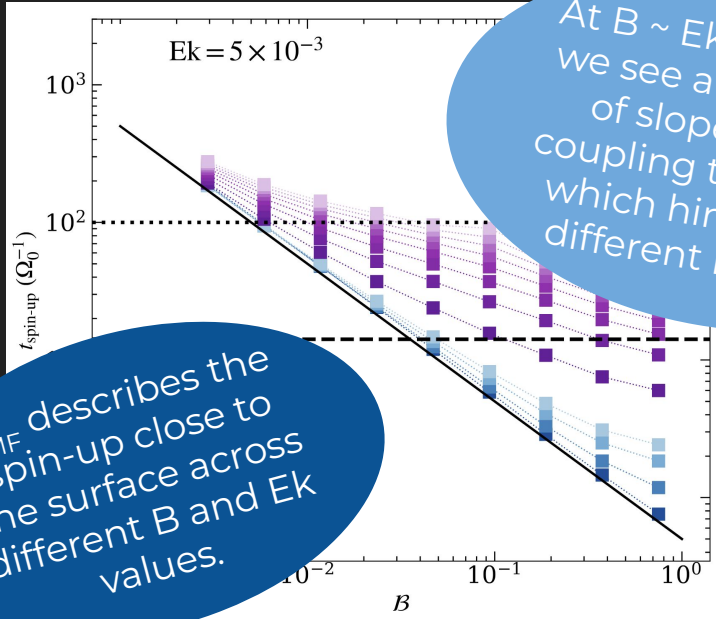
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$$t_{\text{spin-up}}(r) \approx C\tau_{\text{Ek}}^\alpha(\Delta R)\mathcal{B}^{-\gamma}$$

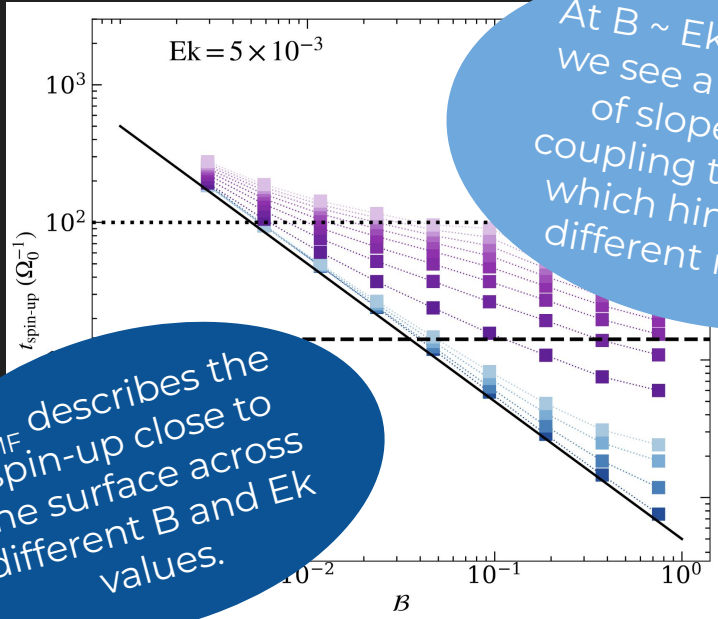


τ_{MF} describes the spin-up close to the surface across different B and Ek values.

Spinning up a two-component fluid III

- To extract the spin-up timescale, we fit

$$\Delta\Omega_s = \varepsilon(1 - e^{-t/t_{\text{spin-up}}})$$

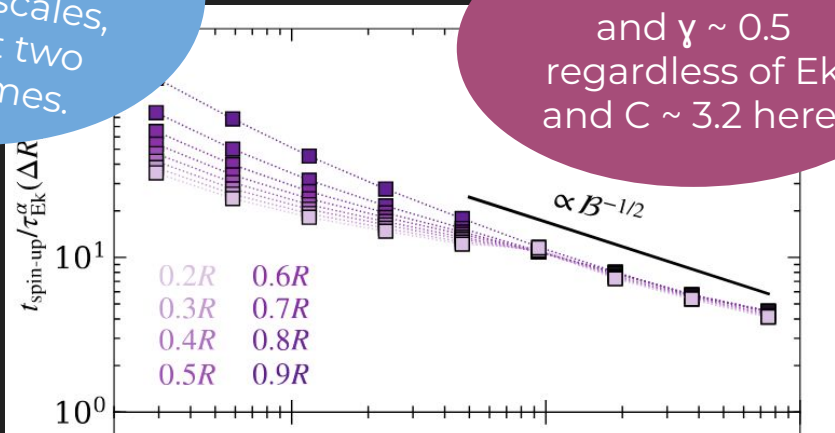


At $B \sim Ek^{0.5}$ ($B \sim 0.07$), we see a separation of slopes in our coupling timescales, which hints at two different regimes.

τ_{MF} describes the spin-up close to the surface across different B and Ek values.

$$t_{\text{spin-up}}(r) \approx C\tau_{Ek}^\alpha (\Delta R)\mathcal{B}^{-\gamma}$$

We find $\alpha \sim 0.85$ and $\gamma \sim 0.5$ regardless of Ek and $C \sim 3.2$ here.





Conclusions & Outlook

Studying the shape of pulsar glitches, provides information on the hidden NS interior.

The spin-up of the outer SF layers is dominated by the mutual friction timescale. Inner layers show more complex behaviour.

We focused on the response of the spherical, two-component NS core following a glitch for the first time.



Conclusions & Outlook

Studying the shape of pulsar glitches, provides information on the hidden NS interior.

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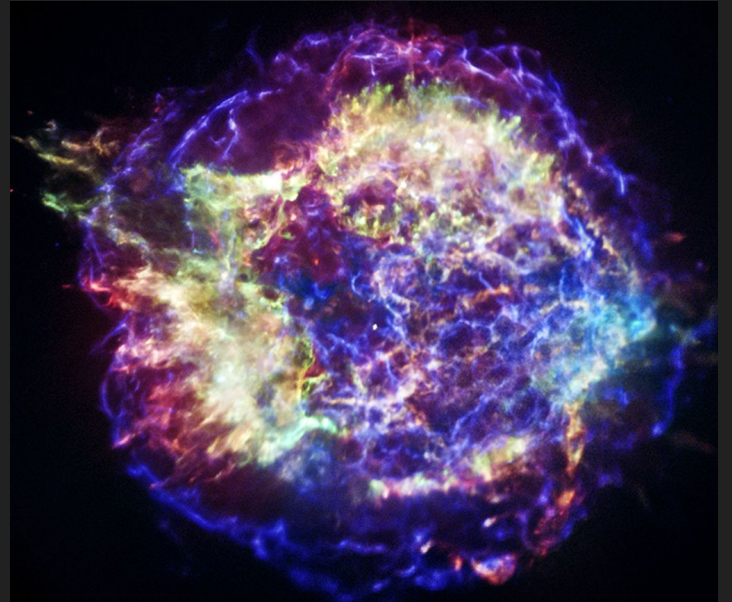
We need to improve our treatment of the crust & study its response.

We still don't fully understand the coupling dynamics for low B values.

We focused on the response of the spherical, two-component NS core following a glitch for the first time.

Our simplified HVBK model neglects the presence of magnetic fields and non-constant densities.

THANK YOU



Cassiopeia A supernova remnant
(credit: NASA/CXC/SAO)

Some advertisement:

I will be joining **Royal Holloway** (University of London)'s growing Astronomy group in October on a Future Leaders Fellowship.

Positions open in the next few years on modelling **pulsar glitches and machine learning**:

2 PhD positions
3 Postdocs

Keep an eye open or come and talk to me if you want to learn more!



**ROYAL
HOLLOWAY**
UNIVERSITY
OF LONDON



**UK Research
and Innovation**

Superfluid Helium-4

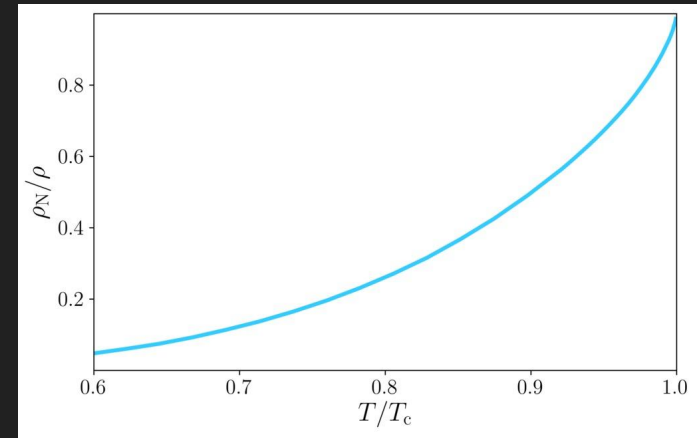
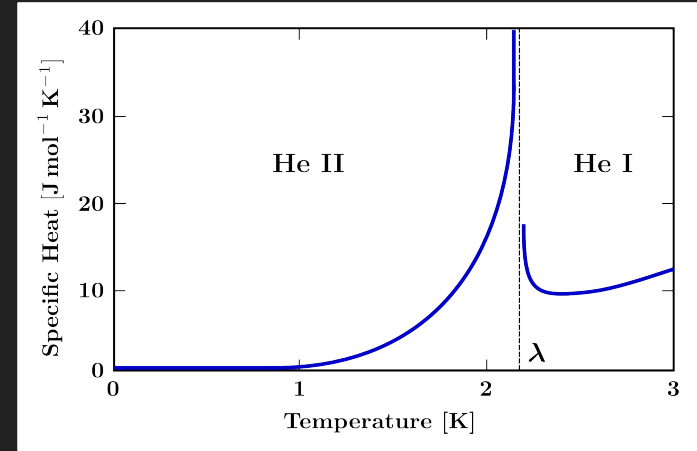
- At low temperatures, helium-4 does not solidify but instead enters a new fluid phase.

The heat capacity resembles the Greek letter λ

Helium II behaviour has been explained by the two-fluid model.

Normal component: viscous properties and heat transport.

Inviscid component: SF features



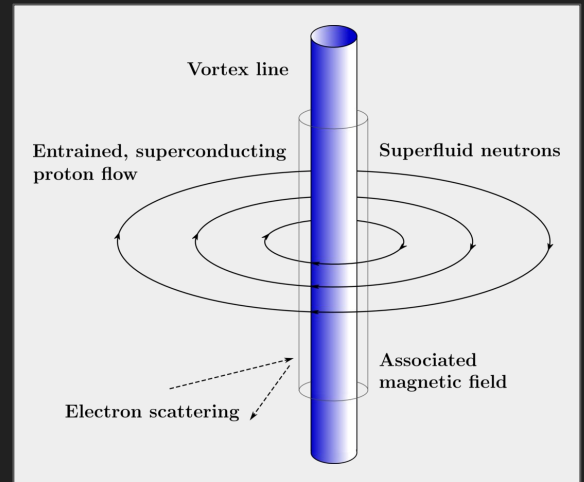
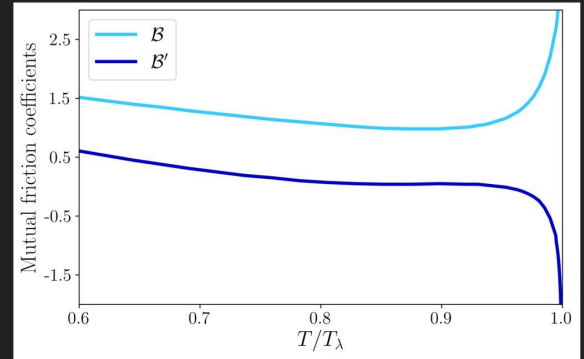
Mutual friction

- Although superfluids flow without friction, they can experience friction as a result of vortices interacting with their surroundings.

$$\mathbf{F}_{\text{mf}} = \mathcal{B}_{\text{He}} \frac{\rho_S \rho_N}{2\rho} \hat{\boldsymbol{\omega}} \times \left[\boldsymbol{\omega} \times (\mathbf{v}_S - \mathbf{v}_N) - \frac{\mathbf{T}}{\rho_S} \right] + \mathcal{B}'_{\text{He}} \frac{\rho_S \rho_N}{2\rho} \left[\boldsymbol{\omega} \times (\mathbf{v}_S - \mathbf{v}_N) - \frac{\mathbf{T}}{\rho_S} \right]$$

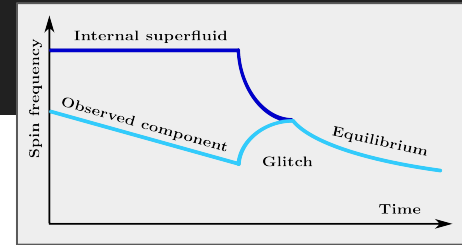
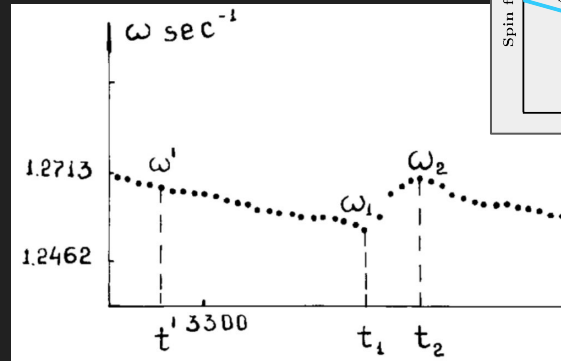
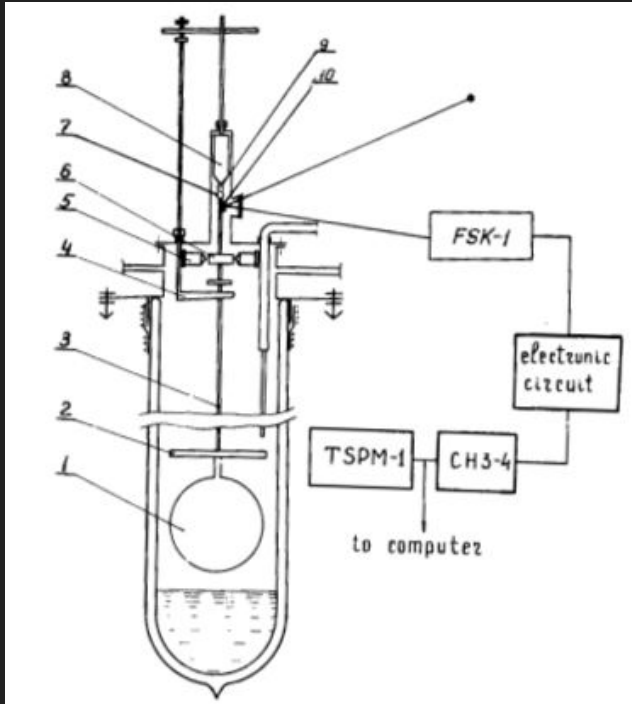
The two coefficients \mathcal{B} / \mathcal{B}' determine the dissipation strength.

In helium II, the coefficients can be measured. For NSs, they need to be calculated.



Laboratory glitches

- In the 1970s, Tsakadze and Tsakadze performed the first (and only!!) systematic study of spin-up in helium II.

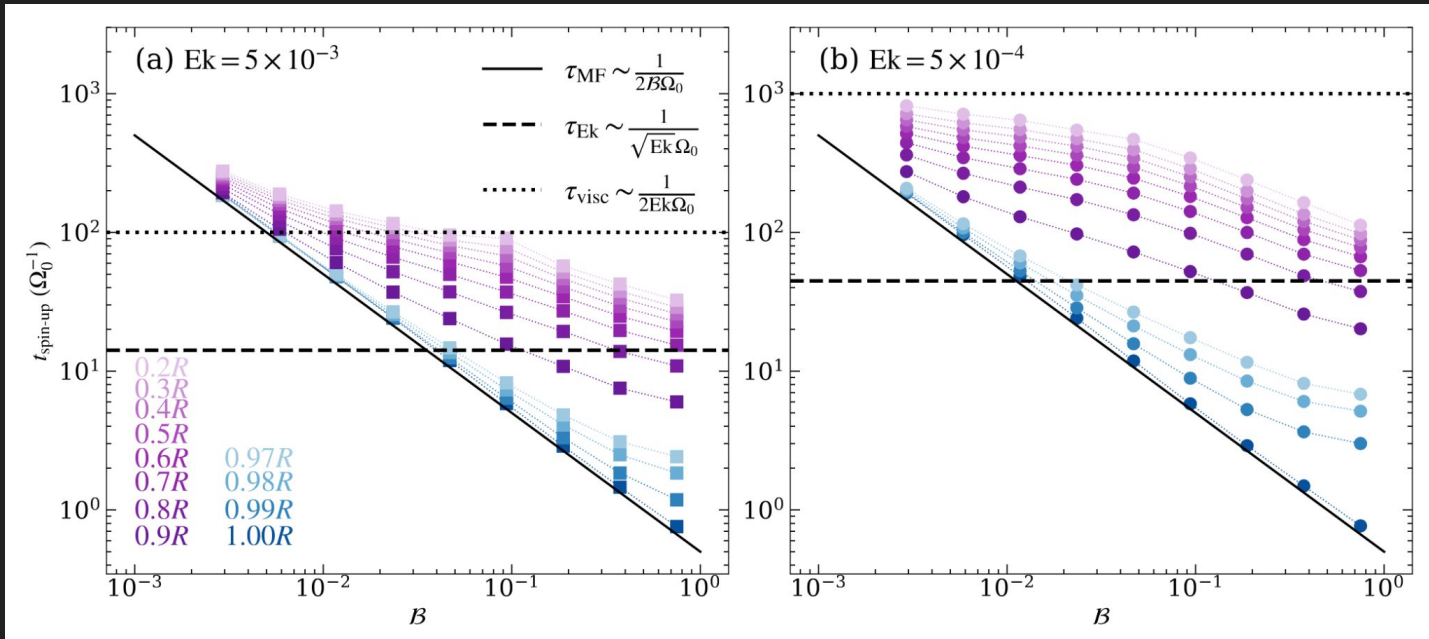


With their basic set up, they might have accidentally observed a glitch.

Credit: Tsakadze and Tsakadze (1980)

Spinning up a two-component fluid IV

- Spin-up timescales for two different Ekman numbers:



Spinning up a two-component fluid V

- Spin-up timescales for two different Ekman numbers:

