Physics of superfluid neutron stars

- growth of the superconducting phase

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Background — Superfluid components

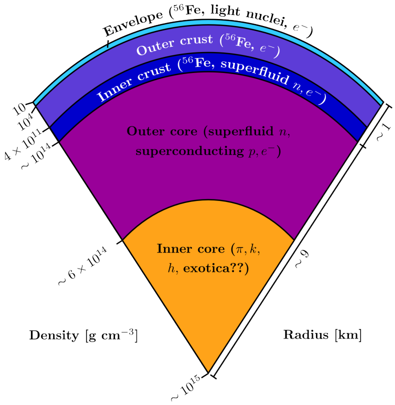


Figure 1: Layered neutron star structure.

- Equilibrium neutron stars with $10^6 - 10^8$ K are cold enough to contain **superfluid** neutrons and **superconducting** protons.
- Cooper pair formation occurs due to an **attractive contribution** to the nucleon-nucleon interaction.
- **Observations** support the presence of these macroscopic quantum states.

How do superfluid components affect the star, i.e. its magnetism?

Superconductivity I

- For $T > T_{\rm cp}$ the matter is normal and the electrical conductivity is dominated by relativistic electrons. Magnetic field dissipation occurs via **standard Ohmic diffusion** on a timescale

$$\tau_{\rm Ohm} = L^2 \frac{4\pi\sigma_{\rm e}}{c^2} \approx 4.3 \times 10^{13} \left(\frac{L}{10^6\,{\rm cm}}\right)^2 \left(\frac{\sigma_{\rm e}}{10^{29}\,{\rm s}^{-1}}\right)^2 {\rm yr}. \quad (1)$$

- Baym, Pethick & Pines (1969) state that for $T < T_{\rm cp}$ the time to **expel magnetic flux** from an initially normal region is

$$\tau_{\rm nucl} \sim \tau_{\rm Ohm} \left(\frac{B_0}{H_{\rm c}}\right)^2 \approx 10^7 \left(\frac{\tau_{\rm Ohm}}{10^{13}\,{\rm yr}}\right) \left(\frac{B_0/H_{\rm c}}{10^{-3}}\right)^2 {\rm yr}. \quad (2)$$

- This implies that the superconducting phase transition has to occur at **constant flux** and essentially no core field evolution takes place.

Superconductivity II

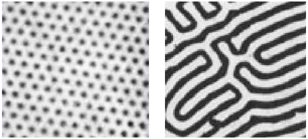


Figure 2: Superconducting states.

- Magnetic flux can be retained in form of a type-II fluxtube state or an intermediate type-I state. The exact phase depends on the characteristic lengthscales involved:

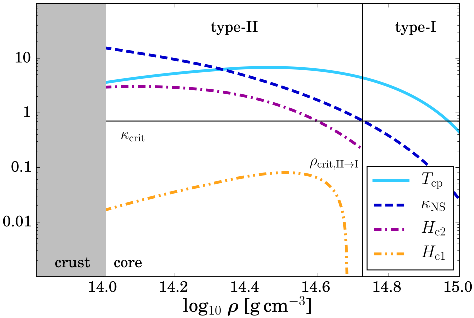
$$\kappa = \frac{\lambda}{\xi_{\rm ft}} \approx 3 \left(\frac{m_{\rm p}^*}{m}\right)^{\frac{3}{2}} \rho_{14}^{-\frac{5}{6}} \left(\frac{x_{\rm p}}{0.05}\right)^{-\frac{5}{6}} \left(\frac{T_{\rm cp}}{10^9\,{\rm K}}\right) > \frac{1}{\sqrt{2}}. \tag{3}$$

- Estimates predict a **type-II state** in the outer core with

$$H_{\rm c1} = \frac{4\pi\mathcal{E}_{\rm ft}}{\phi_0} \approx 1.9 \times 10^{14} \left(\frac{m}{m_{\rm p}^*}\right) \rho_{14} \left(\frac{x_{\rm p}}{0.05}\right) \,{\rm G}, \tag{4}$$

$$H_{\rm c2} = \frac{\phi_0}{2\pi\xi_{\rm ft}^2} \approx 2.1 \times 10^{15} \left(\frac{m_{\rm p}^*}{m}\right)^2 \rho_{14}^{-\frac{2}{3}} \left(\frac{x_{\rm p}}{0.05}\right)^{-\frac{2}{3}} \left(\frac{T_{\rm cp}}{10^9\,{\rm K}}\right)^2 \,{\rm G}. \tag{5}$$

Figure 3: Density-dependent parameters of NS superconductivity calculated for the NRAPR effective equation of state (Steiner et al., 2005). T_{cp} is obtained from Ho, Glampedakis & Andersson (2012).



- Parameters of superconductivity are dependent on the neutron star density, i.e. the **equation of state**.
- At higher densities one eventually has $\kappa < 1/\sqrt{2}$, so that the type-II state should transition into a **type-I state**. The critical density is

$$\rho_{crit,II\to I} \approx 6.4 \times 10^{14} \left(\frac{m_p^*}{m}\right)^{-\frac{9}{5}} \left(\frac{0.05}{x_p}\right) \left(\frac{T_{cp}}{10^9\,K}\right)^{-\frac{6}{5}} \text{g cm}^{-3}. \qquad (6)$$

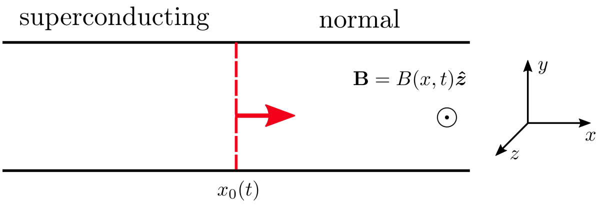
Nucleating superconductivity

- **Magnetohydrodynamical models** of superconducting neutron stars often invoke a type-II fluxtube state to simplify the treatment.
- Models generally discuss the equilibrium case but are not concerned with the **dynamical onset** of the phase transition.
- How is the superconducting state formed as the star cools down? What does the **magnetic flux distribution** look like?

How does superconductivity nucleate?

Where does $\tau_{\mathrm{nucl}} \sim \tau_{\mathrm{Ohm}} \left(\frac{B_0}{H_c}\right)^2$ come from?

Set-up (Baym, 1970)



- The superconducting region expands into the normal region: The interface is positioned at $x_0(t)$ and moves with a velocity $v(t) = dx_0(t)/dt$.
- The field $\boldsymbol{B} = B(x,t)\hat{\boldsymbol{z}}$ in the normal region satisfies standard Maxwell equations, which combined with Ohm's law give a **diffusion equation**

$$\frac{\partial^2 B(x,t)}{\partial x^2} = \frac{4\pi\sigma}{c^2}\frac{\partial B(x,t)}{\partial t} \equiv \frac{1}{D}\frac{\partial B(x,t)}{\partial t}. \quad (7)$$

Diffusive solution I

- Solve the diffusion equation with appropriate **boundary conditions**. **Thermodynamics** dictate the conditions at the interface:

$$\lim_{x\to\infty} B(x,t) \to B_0, \qquad B(x_0(t),t) = H_c, \qquad \left.\frac{\partial B}{\partial x}\right|_{x=x_0(t)} = -\frac{H_c}{D}\,v(t). \qquad (8)$$

- The problem can be solved by taking into account the **self-similarity**, i.e. the absence of a characteristic lengthscale. In this case, we obtain an implicit time-dependence of the form $B(x,t) = B(y)$ where $y \equiv x/x_0(t)$.

- Follow Pippard (1950) and use the ansatz $B(y) = H_c[1 + f(y)]$ to obtain

$$\frac{d^2 f}{dy^2} + \frac{x_0(t)v(t)}{D}\,y\frac{df}{dy} = \frac{d^2 f}{dy^2} + \alpha y\frac{df}{dy} = 0. \qquad (9)$$

Diffusive solution II

- The definition of α relates the interface position and its velocity. Both quantities are thus **not independent** but have to satisfy

$$\frac{x_0(t)v(t)}{D} = \alpha \qquad \Rightarrow \qquad \frac{dx_0^2}{dt} = 2D\alpha. \tag{10}$$

- Integration leads to the following under the assumption that $x_0(0) = 0$

$$x_0(t) = (2D\alpha t + c_1)^{\frac{1}{2}} = (2D\alpha t)^{\frac{1}{2}} . \tag{11}$$

- We find an **analytical solution** in the limit $B_0 \ll H_c$:

$$B(x,t) = H_c \left(1 - \int_0^{\alpha\left[\frac{x}{x_0(t)} - 1\right]} d\xi \, e^{-\left[\xi + \xi^2 \frac{B_0}{2H_c}\right]} \right) . \tag{12}$$

Timescale

- The exact value for α is determined by the **third boundary condition**:

$$x_0(t) = \left(\frac{H_c}{B_0} 2Dt\right)^{\frac{1}{2}}, \qquad v(t) = \left(\frac{H_c}{B_0} \frac{D}{2t}\right)^{\frac{1}{2}}. \qquad (13)$$

- The time τ_{nucl} to move the interface a distance L into the normal conducting phase (expel magnetic flux), is

$$\tau_{\mathrm{nucl}} = \frac{L^2}{D} \frac{B_0}{2H_c} = \tau_{\mathrm{ohm}} \frac{B_0}{2H_c}$$

$$\sim 10^{10} \left(\frac{\tau_{\mathrm{Ohm}}}{10^{13}\,\mathrm{yr}}\right) \left(\frac{B_0/H_c}{10^{-3}}\right)^2 \mathrm{yr}. \qquad (14)$$

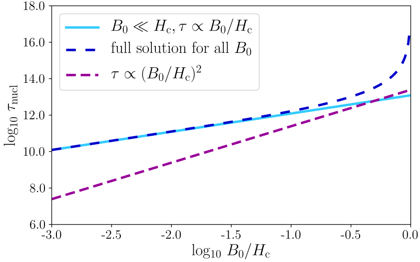


Figure 4: Nucleation timescales as a function of B_0/H_c for $\sigma_e = 5.5 \times 10^{28}$ s and $L = 10^6$ cm.

Nucleation vs. Cooling I

Nucleation timescale is very long. Can flux expulsion play any role?

- The **transition temperature** $T_{\rm cp}$ is not constant throughout the neutron star's interior but **density-dependent**.
- The superconducting region expands inside the star as it cools down. Its size depends on the equation of state and pairing gap model.

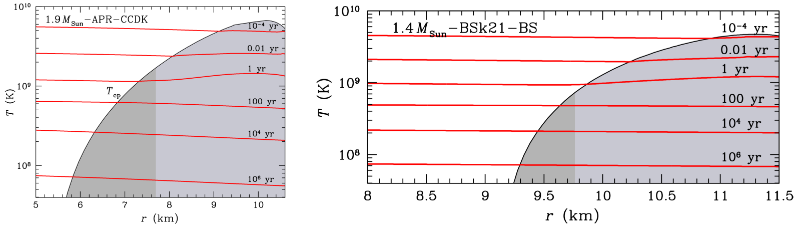


Figure 5: Two gap models, EoSs with mUrca and p pairing cooling (Ho, Andersson & Graber (2017) submitted).

Nucleation vs. Cooling II

- Flux can only be expelled if cooling proceeds slower than nucleation.
- Alternatively, compare the **lengthscales** of cooling and flux expulsion.

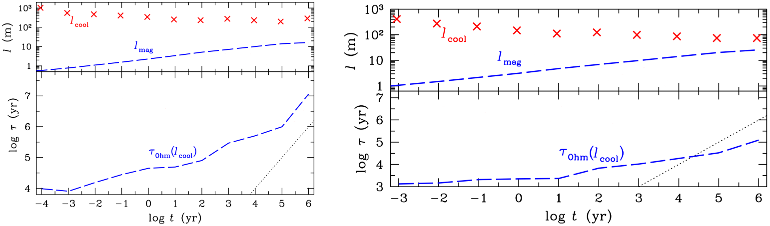


Figure 6: Lengthscale comparison for $B_0/H_c \sim 10^{-5}$ (Ho, Andersson & Graber (2017) submitted).

- For **broad gap models**, cooling is always faster than flux expulsion. Only for **very narrow gaps** and low magnetic fields $B_0 \sim 10^{10}$ G it might be possible to create a flux-free spherical shell after $t \sim 10^5$ yr.

Diffusion cannot drive flux expulsion from the entire neutron star core, but macroscopic flux-free Meissner regions $\sim 1 - 10\,$m could exist.

- How would this look like in a more realistic three dimensional scenario?
- Solve the diffusion problem for a **spherical superconducting seed region** in axisymmetry and study its growth by deriving a solution for the induction $\boldsymbol{B}(r, \theta, t) = f_r(r, t)\, h_r(\theta)\hat{\boldsymbol{r}} + f_\theta(r, t)\, h_\theta(\theta)\hat{\boldsymbol{\theta}}$ in normal matter.
- The angular functions are determined by **Legendre polynomials**, whereas the radial contributions are obtained by solving the equations

$$\frac{d^2 f_r}{dy^2} + \left(\frac{4}{y} + \alpha y\right)\frac{df_r}{dy} = 0, \qquad f_\theta = f_r + \frac{y}{2}\frac{\partial f_r}{\partial y}, \tag{15}$$

under appropriate boundary conditions.

Next steps

- In the limit $B_0 \ll H_c$, a bit of algebra leads to the expulsion **timescale**

$$\tau_{\mathrm{nucl,3D}} = \frac{L^2}{D} \frac{B_0}{2\sqrt{6} H_c} \approx 0.4 \tau_{\mathrm{nucl,1D}}. \tag{16}$$

- The **expansion** of the spherical interface is likely to become **unstable**. At what radius does this happen and could the perturbation of the interface lead to shorter flux expulsion timescales?

Modelling the actual structure of the superconducting state through the phase transition should require a Ginzburg-Landau approach as is done for laboratory superconductors.


Thank you.
Any questions?

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