ROYAL HOLLOWAY UNIVERSITY OF LONDON

UK QF Webinar,
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Simulating sudden hiccups of superfluid neutron stars in 3D

arXiv:2407.18810

Dr Vanessa Graber
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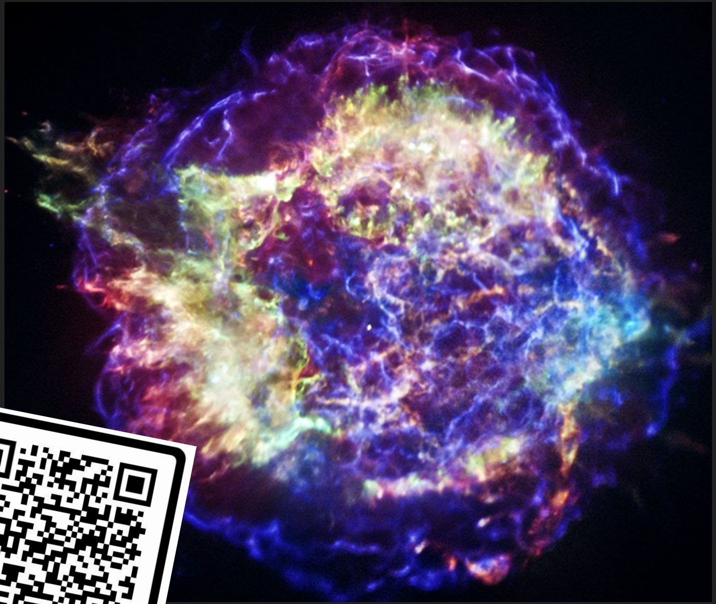
in collaboration with J. Rafael Fuentes
(University of Colorado Boulder)

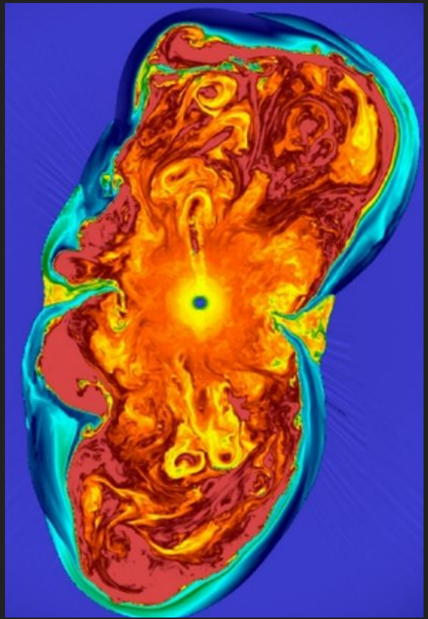


SCAN ME

Cassiopeia A
supernova remnant
(credit: NASA/CXC/SAO)

Neutron star formation

- NSs are one of three types of **compact remnants**, created at the end of stellar evolution.
- When massive stars of 8 - 25 solar masses run out of fuel, they collapse under their own gravitational attraction and explodes in supernovae.
- During the collapse, **electron capture** processes ($p + e^- \rightarrow n + \nu_e$) produce (a lot of) neutrons.



Snapshot of a 3D core-collapse supernova simulation (Mösta et al., 2014).

mass: 1.2 - 2.1 $M_\odot$

radius: 9 - 15 km

density: 10^{15} g/cm^3

period: 10ms - 10s

B-field: 10^8 - 10^{15}G

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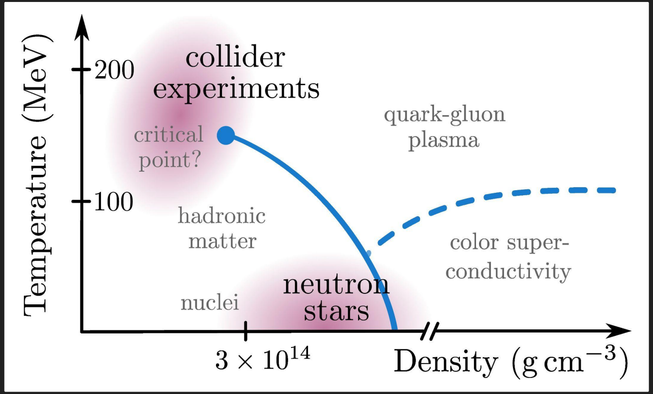
Neutron star interiors

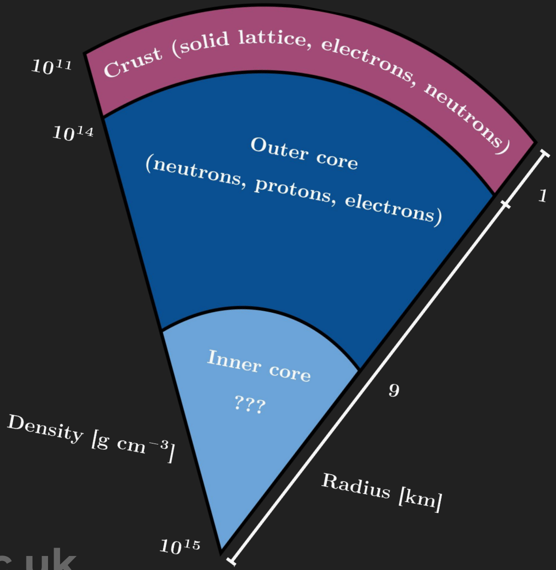
- The interior structure of neutron stars is complex and influenced by the (un-known) nuclear matter equation of state.



After ~10^4 yrs interior tem-peratures reach ~10^6 - 10^8 K.

Their structure is layered very much like the Earth.

We can decompose these stars into solid crusts and fluid cores.



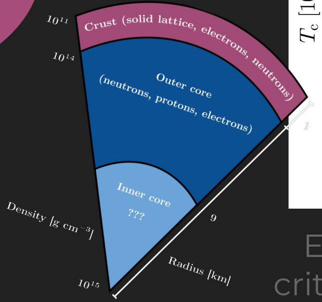
Superfluid components

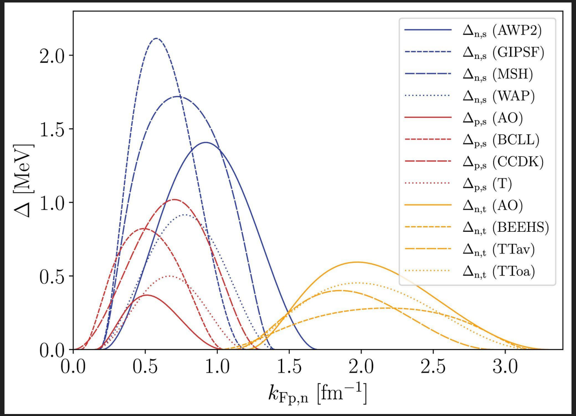
- Although neutron stars are hot compared to laboratory experiments, they are cold in terms of their extremely high densities.

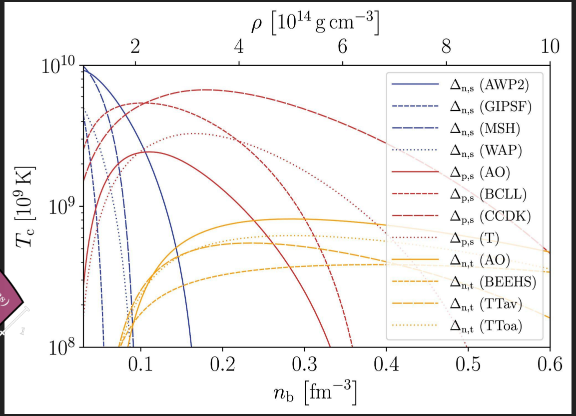
Interiors are well below neutron and proton Fermi temperatures (~10^{12} K).

Below a critical temperature T_c the fermionic nucleons form Cooper pairs.

Neutron stars contain at least 3 superfluid components.







Energy gaps and corresponding critical temperatures for the NRAPR EoS (Ascenzi, Graber & Rea, 2024).

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Quantum vortices

Waterspout off the Florida coast (Credit: NOAA Photo Library).

- Superfluids are macroscopic quantum states, characterised by a wave function $\Psi=\Psi_0 e^{i\varphi}$, which satisfies the Schrödinger equation.

$\mathbf{v}_{SF} = \hbar/m_c \nabla \varphi$ dictates $\boldsymbol{\omega} = \nabla \times \mathbf{v}_{SF} = 0$. Superflow is irrotational.

SFs rotate by forming quantised vortices.

Each vortex carries a quantum of circulation $\kappa = h/m_c$.

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Quantum vortices

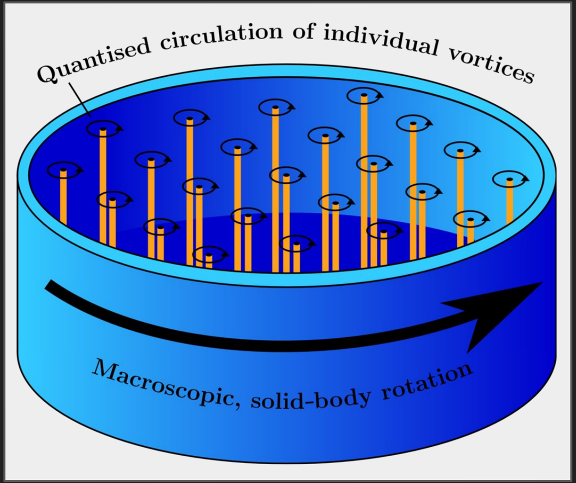

Illustration of superfluid rotation (Graber et al., 2017).

$$\kappa = \frac{h}{2m_{\rm n}} \approx 2.0 \times 10^{-3}\,{\rm cm^2\,s^{-1}},$$

$$\mathcal{N}_{\rm n} = \frac{2\Omega}{\kappa} = \frac{4\pi}{\kappa P} \approx 6.3 \times 10^5 P_{10}^{-1}\,{\rm cm^{-2}},$$

- Superfluids are macroscopic quantum states, characterised by a wave function $\Psi = \Psi_0 e^{i\varphi}$, which satisfies the Schrödinger equation.

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SFs rotate by forming quantised vortices.

Each vortex carries a quantum of circulation $\kappa = h/m_c$.

- The vortices form an array that mimics solid-body rotation on large scales $\omega = 2\Omega = N_n \kappa$.

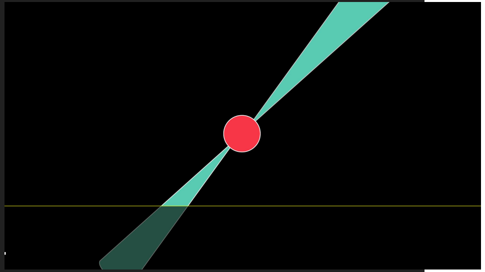
Observing neutron stars

period: 10ms - 10s

B-field: 10^8 - 10^{15}G

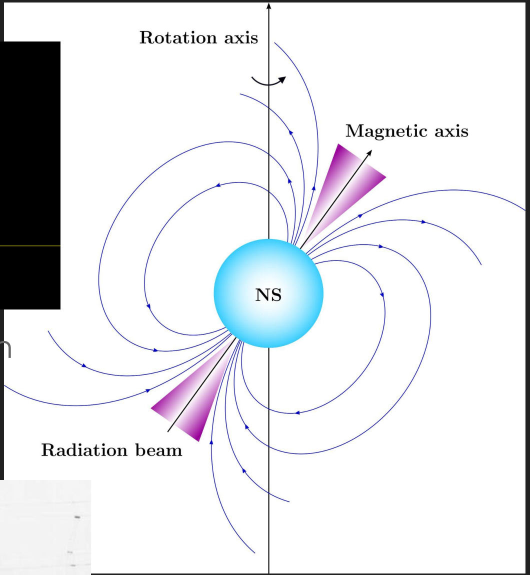
- Because the rotation and magnetic axes are misaligned, neutron stars **emit radio beams like a lighthouse**.

- These pulses can be observed (timed) with radio telescopes. This is how neutron stars were first detected and why we call them **pulsars**.



Credit: J. Christiansen

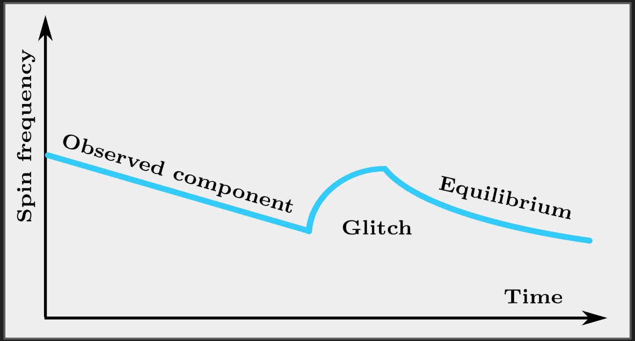
Sketch of the neutron-star exterior.



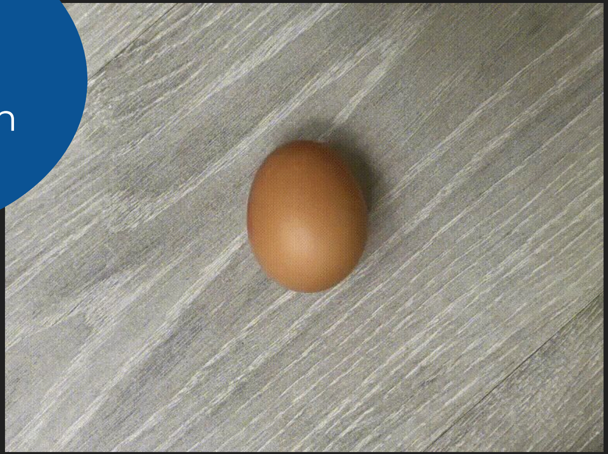
Dame Jocelyn Bell Burnell in front of her radio telescope in Cambridge, UK.

Pulsar glitches I

- We observe through pulsar timing that the regular spin-down of neutron stars can be interrupted by sudden hiccups.



We can understand the glitch origin with an experiment.



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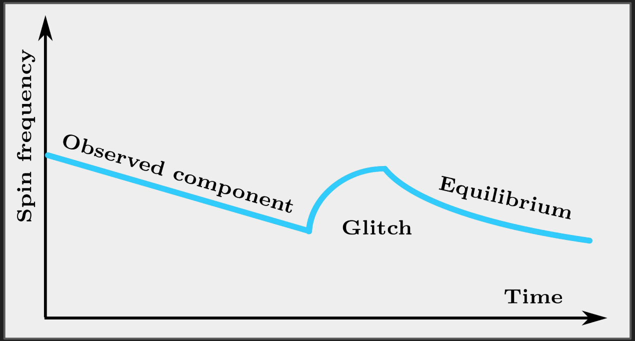

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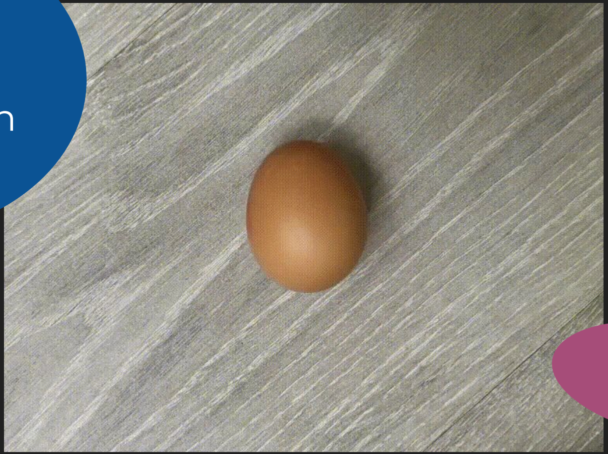

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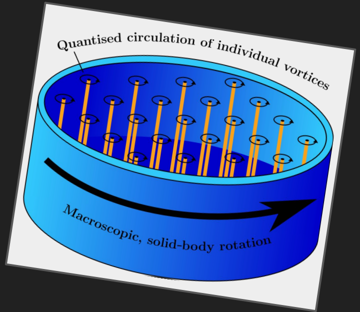
Pular glitches II

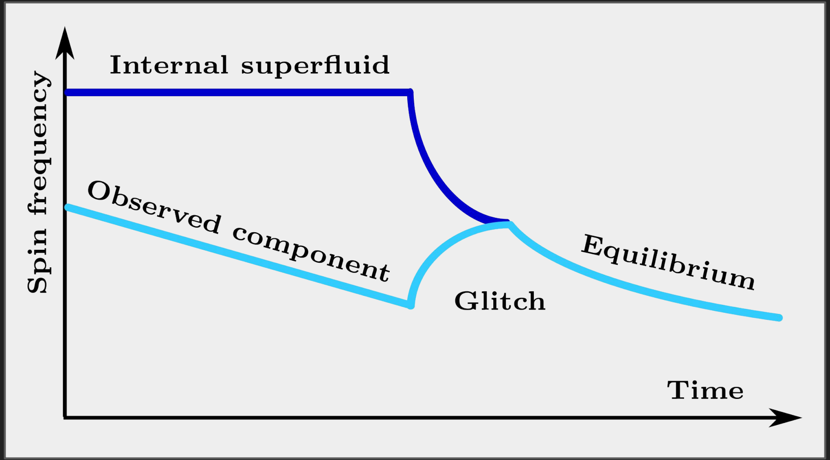
- Spin-up glitches can be naturally explained in a multi-component neutron star model.



Superfluid spin-down can be impeded by pinning of vortices to crustal lattice.

The crustal superfluid acts as a reservoir of angular momentum.

The shape of the glitch encodes the (hidden) internal neutron star physics.



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A problem at three different scales

- To model the pulsar glitch phenomenon in its entirety, we need to understand (complex) physics at three different scales:

Credit: Johann Siemens, John Price, Geranimo

Individual vortices

See, e.g., Epstein & Baym (1992), Jones (1992), Wlazłowski et al. (2016), Marmorini et al. (2024)

O(100) vortices

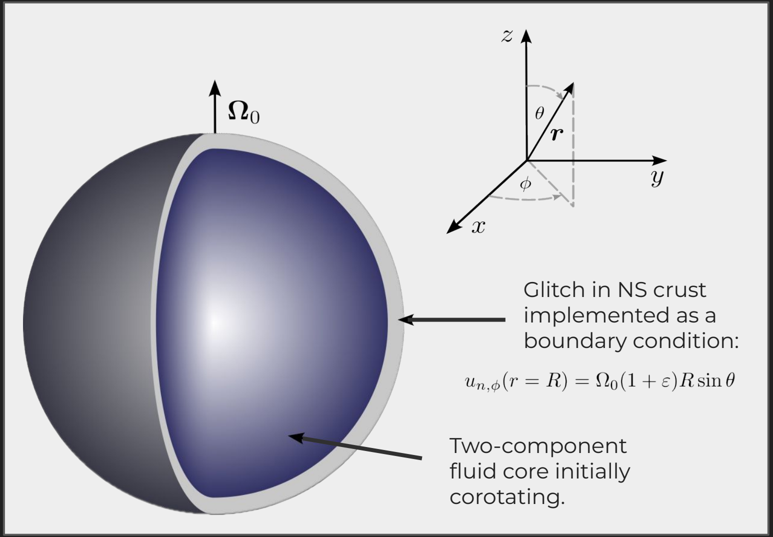
See, e.g., Warszawski & Melatos (2011), Drummond & Melatos (2017, 2018), Howitt et al. (2020), Liu et al. (2024)

O(>10^{10}) vortices

See, e.g., Peralta et al. (2005) Haskell et al. (2012), van Eysden & Melatos (2013), Sourie et al. (2020), Graber et al. (2018)

Numerical experiment set-up

- In our new study, we focus on the response of a two-component fluid core to a glitch that is driven by the crustal superfluid.



Fuentes & Graber (2024)

We model the NS crust as an infinitely thin boundary.

The core is modelled as superfluid (n) and a viscous (p and e^-) mixture.

At t=0, we increase the rotation rate of the viscous fluid at the boundary to $\Omega_0(1+\varepsilon)$ with $\varepsilon \sim 10^{-3}$.

Goal: Study the core's response to the glitch in full sphere for the first time.

HVBK Equations

- We focus on the hydrodynamical picture and solve the Hall–Vinen–Bekarevich–Khalatnikov (HVBK) equations initially developed for laboratory superfluid helium with the pseudo-spectral code Dedalus:

$$\frac{\partial \boldsymbol{u}_n}{\partial t} + \boldsymbol{u}_n \cdot \boldsymbol{\nabla} \boldsymbol{u}_n = -\boldsymbol{\nabla} \tilde{\mu}_n + \nu \boldsymbol{\nabla}^2 \boldsymbol{u}_n + \frac{\boldsymbol{F}_{\mathrm{MF}}}{\rho_n} , \qquad (1)$$

$$\frac{\partial \boldsymbol{u}_s}{\partial t} + \boldsymbol{u}_s \cdot \boldsymbol{\nabla} \boldsymbol{u}_s = -\boldsymbol{\nabla} \tilde{\mu}_s - \frac{\boldsymbol{F}_{\mathrm{MF}}}{\rho_s} , \qquad (2)$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u}_n = 0 = \boldsymbol{\nabla} \cdot \boldsymbol{u}_s , \qquad (3)$$

$$\boldsymbol{F}_{\mathrm{MF}} = \rho_s \left[\mathcal{B} \left(\hat{\boldsymbol{\omega}}_s \times (\boldsymbol{\omega}_s \times \boldsymbol{u}_{sn}) \right) + \mathcal{B}' \left(\boldsymbol{\omega}_s \times \boldsymbol{u}_{sn} \right) \right] , \qquad (4)$$

with $\boldsymbol{u}_{sn} = \boldsymbol{u}_s - \boldsymbol{u}_n, \ \boldsymbol{\omega}_s = \boldsymbol{\nabla} \times \boldsymbol{u}_s$

$\mathbf{u}_n$ and $\mathbf{u}_s$ denote the two fluid velocities and $\varrho_{n,s}$ the mass densities with $\varrho = \varrho_n + \varrho_s$.

ν is the kinematic viscosity of the normal fluid and $\mu_{n,s}$ the chemical potentials.

$\mathbf{F}_{MF}$ is the mutual friction force due to interactions of vortices and their surroundings.

Characteristic time scales

- For realistic neutron stars, we estimate the mutual friction and Ekman timescale as follows:

$$\tau_{\rm MF} \sim \frac{1}{2\Omega_s \mathcal{B}} \sim 80\ {\rm s} \left(\frac{P_{\rm rot}}{0.1\ {\rm s}}\right) \left(\frac{\mathcal{B}}{10^{-4}}\right)^{-1}$$

$$\tau_{\rm Ek} \sim \frac{1}{\sqrt{\rm Ek}\,\Omega_n}$$ with $${\rm Ek} = \nu / 2\Omega_n R^2$$

$$\tau_{\rm Ek} \sim 10^3\ {\rm s} \left(\frac{x_n}{0.05}\right)^{-3/4} \left(\frac{\rho_n}{10^{14}\ {\rm g\ cm^{-3}}}\right)^{-1/4} \left(\frac{P_{\rm rot}}{0.1\ {\rm s}}\right)^{1/2} \left(\frac{T}{10^8\ {\rm K}}\right) \left(\frac{R}{10^6\ {\rm cm}}\right)$$

For electron-electron scattering, we have ν~10^5cm s^2 and Ek~10^{-9}.

For the scattering of electrons off of neutron vortices, we have B~10^{-4} and B'~B^2.

Note: Due to numerical constraints, we vary B~1-10^{-3} with B/B'~2 and set Ek~5x$10^{-3(4)}$.

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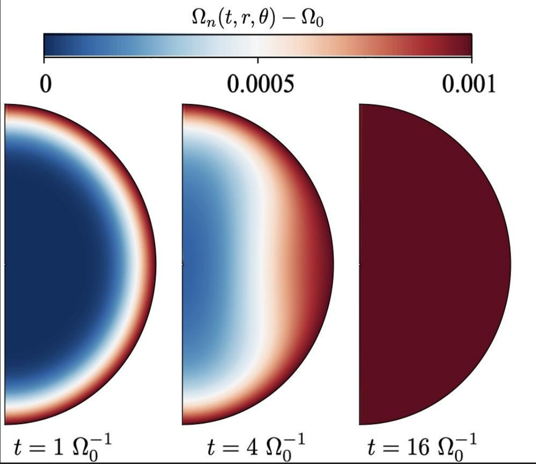
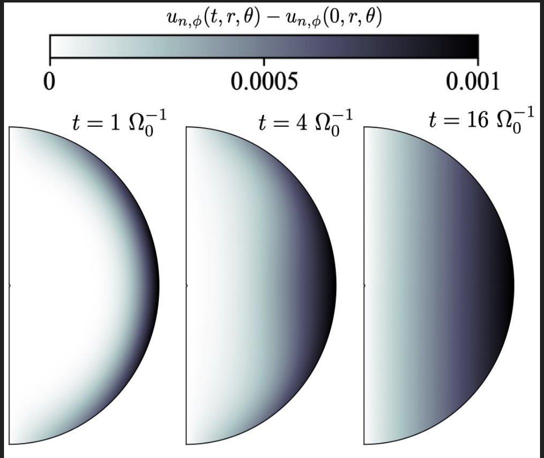
Spinning up a single-component fluid I

- After the sudden spin-up, a thin boundary layer forms just below the crust and causes the bulk fluid to accelerate via Ekman pumping.





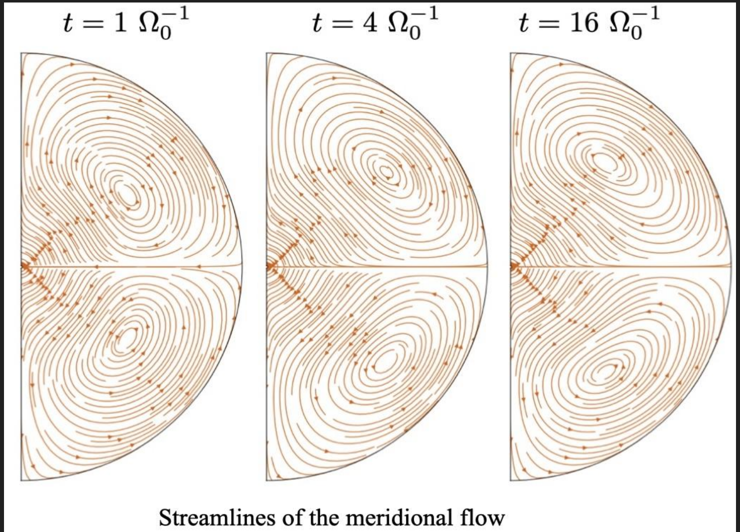
The axisymmetric behaviour is a direct result of the Taylor-Proudman theorem.

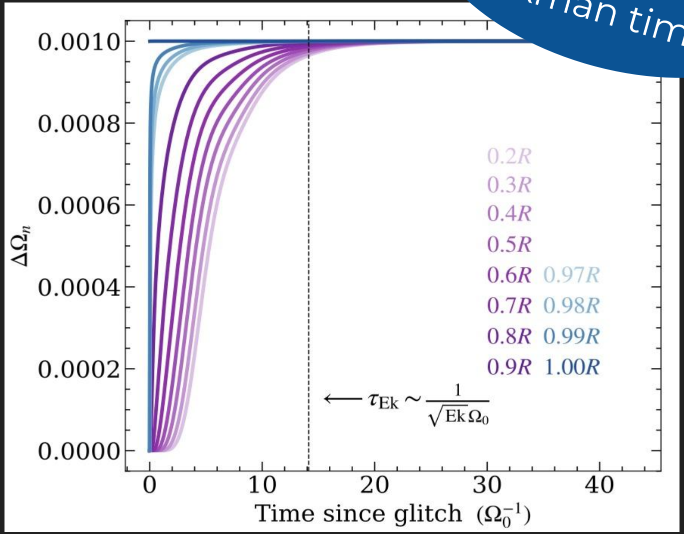
- As the evolution progresses, the azimuthal velocity becomes axisymmetric with constant magnitude over cylindrical surfaces.

Spinning up a single-component fluid II

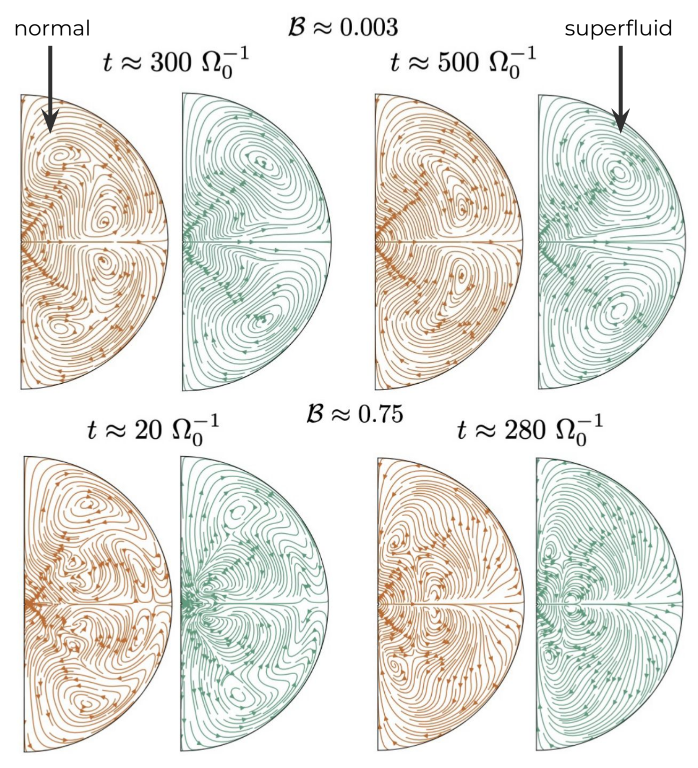
- Ekman pumping leads to the formation of a stable circular flow pattern in each semi-hemisphere.

As expected, a viscous fluid spins up on the Ekman time.





Spinning up a two-component fluid I



- The flow patterns look more complex than for the single-component case and we find similar patterns to earlier works in spherical shells (see Peralta et al. 2005, 2006, 2008).

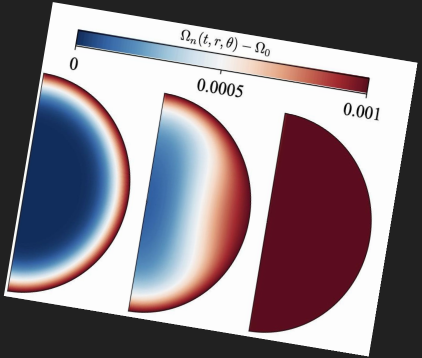
In the weak coupling regime, the fluids seem to evolve almost independently.

For strong coupling, the superfluid follows the viscous fluid pattern.

For B~0.75, the normal fluid no longer develops the single cell flow structure.

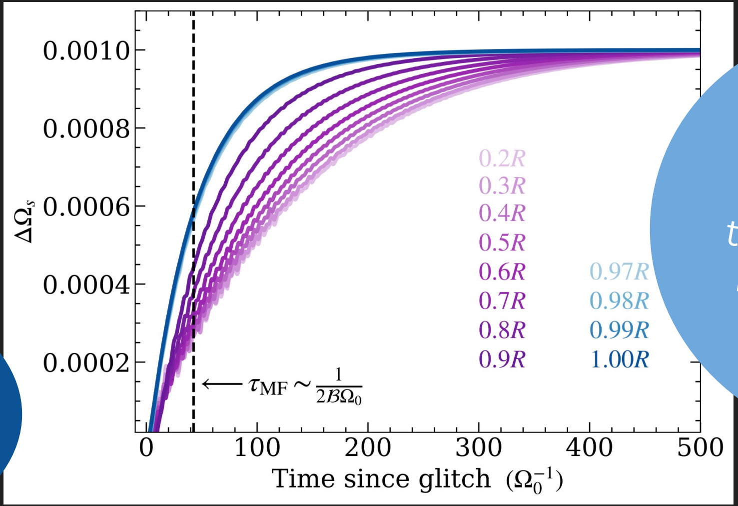
Spinning up a two-component fluid II

- While the overall evolution is qualitatively similar and we obtain constant azimuthal velocities over cylindrical surfaces, the spin-up of the superfluid is delayed because of the mutual friction coupling.



Superfluid can only accelerate once the Ekman pumping has spun up the viscous component.

Once differential rotation builds up, the superfluid is coupled via mutual friction.



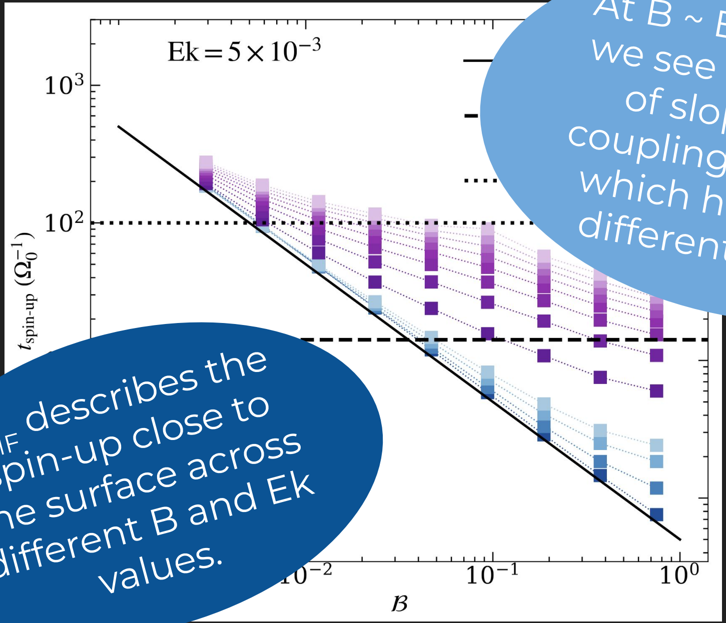
Outer superfluid layers spin up on the mutual friction timescale, while it takes longer for the inner layers.

Spinning up a two-component fluid III

- To extract the spin-up timescale, we fit

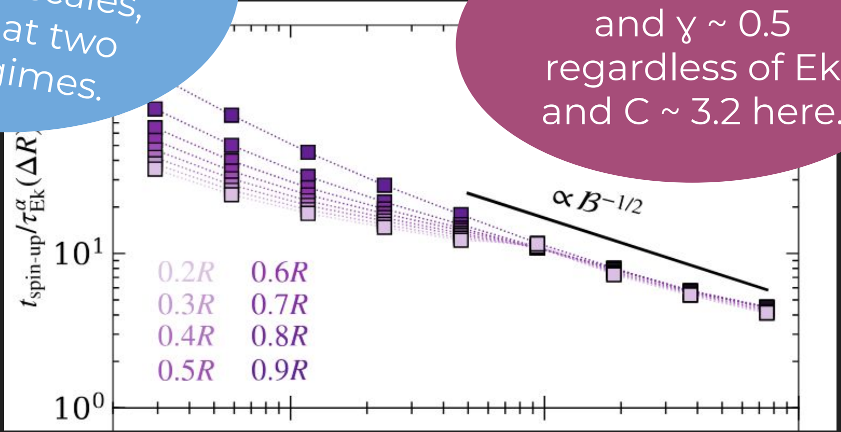
$$\Delta\Omega_s = \varepsilon\left(1 - e^{-t/t_{\text{spin-up}}}\right)$$

$$t_{\text{spin-up}}(r) \approx C\tau_{\text{Ek}}^{\alpha}(\Delta R)\mathcal{B}^{-\gamma}$$



At B ~ Ek$^{0.5}$ (B~0.07), we see a separation of slopes in our coupling timescales, which hints at two different regimes.

τ_{MF} describes the spin-up close to the surface across the different B and Ek values.



We find α ~ 0.85 and γ ~ 0.5 regardless of Ek and C ~ 3.2 here.

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A problem at three different scales

- To model the pulsar glitch phenomenon in its entirety, we need to understand (complex) physics at three different scales:

Credit: Johann Siemens, John Price, Geranimo

Individual vortices

See, e.g., Epstein & Baym (1992), Jones (1992), Wlazłowski et al. (2016), Marmorini et al. (2024)

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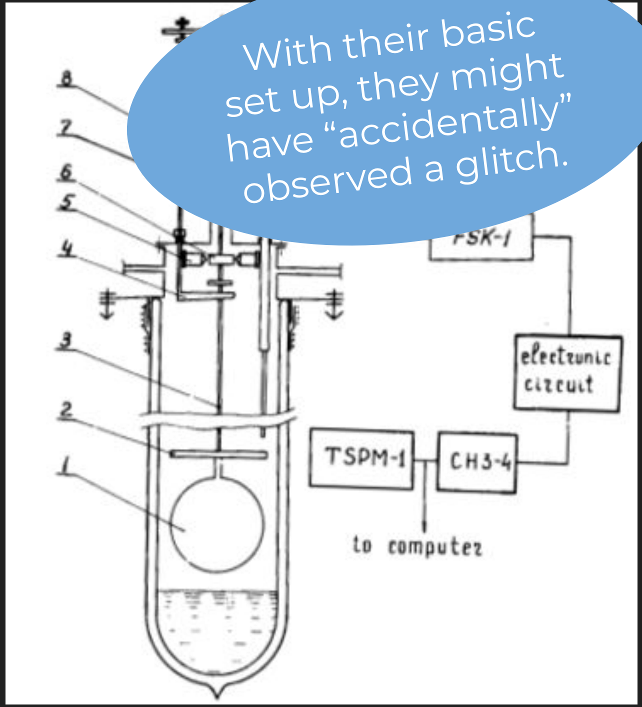
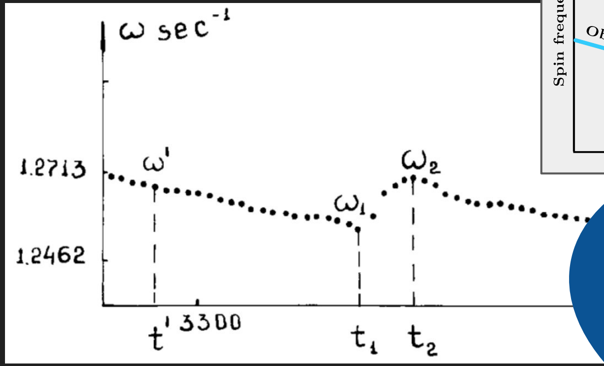
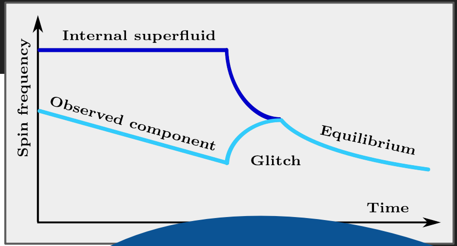
Glitch analogues in the laboratory



With their basic set up, they might have "accidentally" observed a glitch.

- In the 1970s, Tsakadze and Tsakadze performed a systematic study of spin-up in helium II to understand the long-term relaxation observed following glitches.





Laboratory SFs can help us understand the small-scale dynamics that we cannot access in NSs.

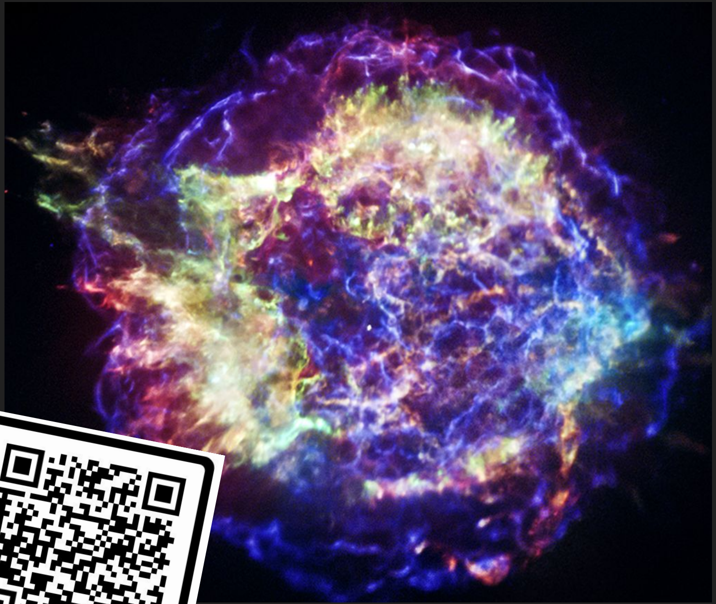
Credit: Tsakadze and Tsakadze (1980)

Conclusions

Studying the shape of pulsar glitches, provides information on the hidden NS interior.

The spin-up of the outer SF layers is dominated by the mutual friction timescale. Inner layers show more complex behaviour.

We focused on the response of the spherical, two-component NS core following a glitch for the first time.



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Cassiopeia A supernova remnant (credit: NASA/CXC/SAO)

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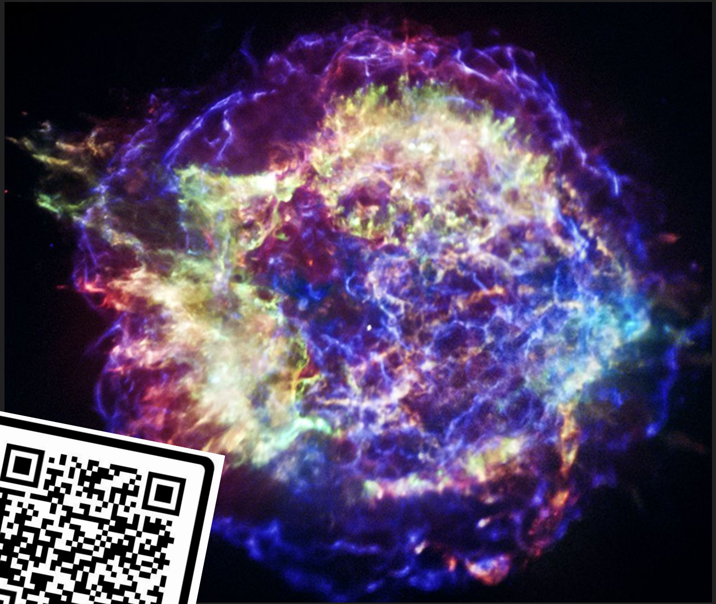
Outlook

We need to improve our treatment of the crust & study its response.

We still don't fully understand the coupling dynamics for low B values.

Our simplified HVBK model neglects the presence of magnetic fields and non-constant densities.

Explore laboratory analogues to better understand vortex dynamics.



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Cassiopeia A supernova remnant (credit: NASA/CXC/SAO)

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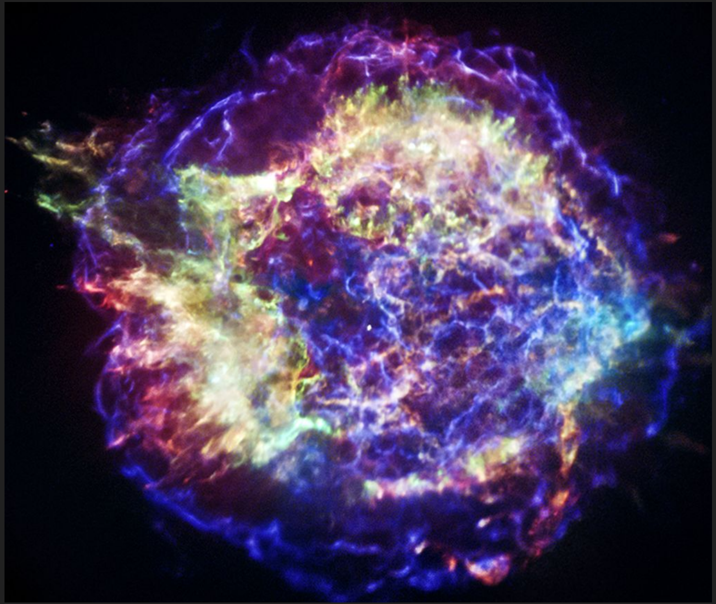
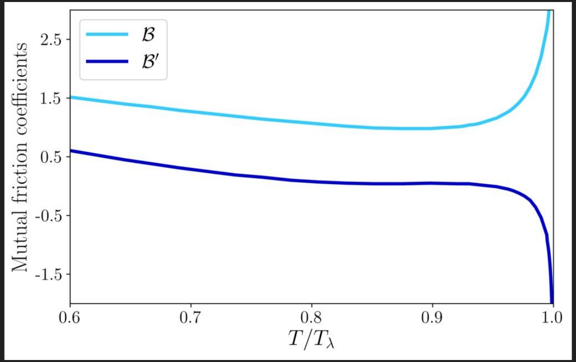
THANK YOU



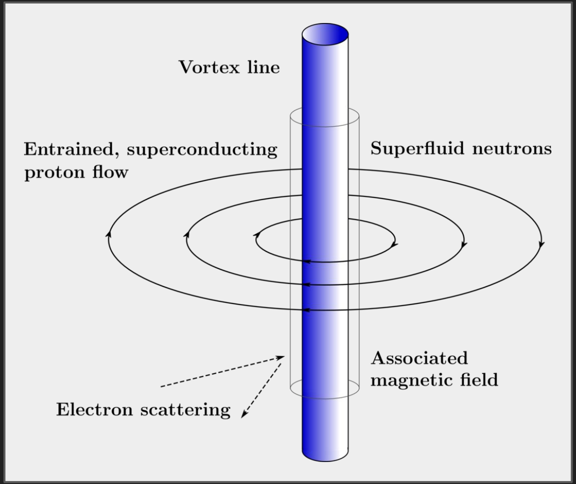
Cassiopeia A
supernova remnant
(credit: NASA/CXC/SAO)

Mutual friction

- Although superfluids flow without friction, they can experience friction as a result of vortices interacting with their surroundings.



$$\mathbf{F}_{\rm mf} = \mathcal{B}_{\rm He} \frac{\rho_{\rm S}\rho_{\rm N}}{2\rho}\,\hat{\boldsymbol{\omega}}\times\left[\boldsymbol{\omega}\times(\mathbf{v}_{\rm S}-\mathbf{v}_{\rm N}) - \frac{\mathbf{T}}{\rho_{\rm S}}\right]$$
$$+\mathcal{B}'_{\rm He}\frac{\rho_{\rm S}\rho_{\rm N}}{2\rho}\left[\boldsymbol{\omega}\times(\mathbf{v}_{\rm S}-\mathbf{v}_{\rm N}) - \frac{\mathbf{T}}{\rho_{\rm S}}\right]$$



The two coefficients B/B' determine the dissipation strength.

In helium II, the coefficients can be measured. For NSs, they need to be calculated.

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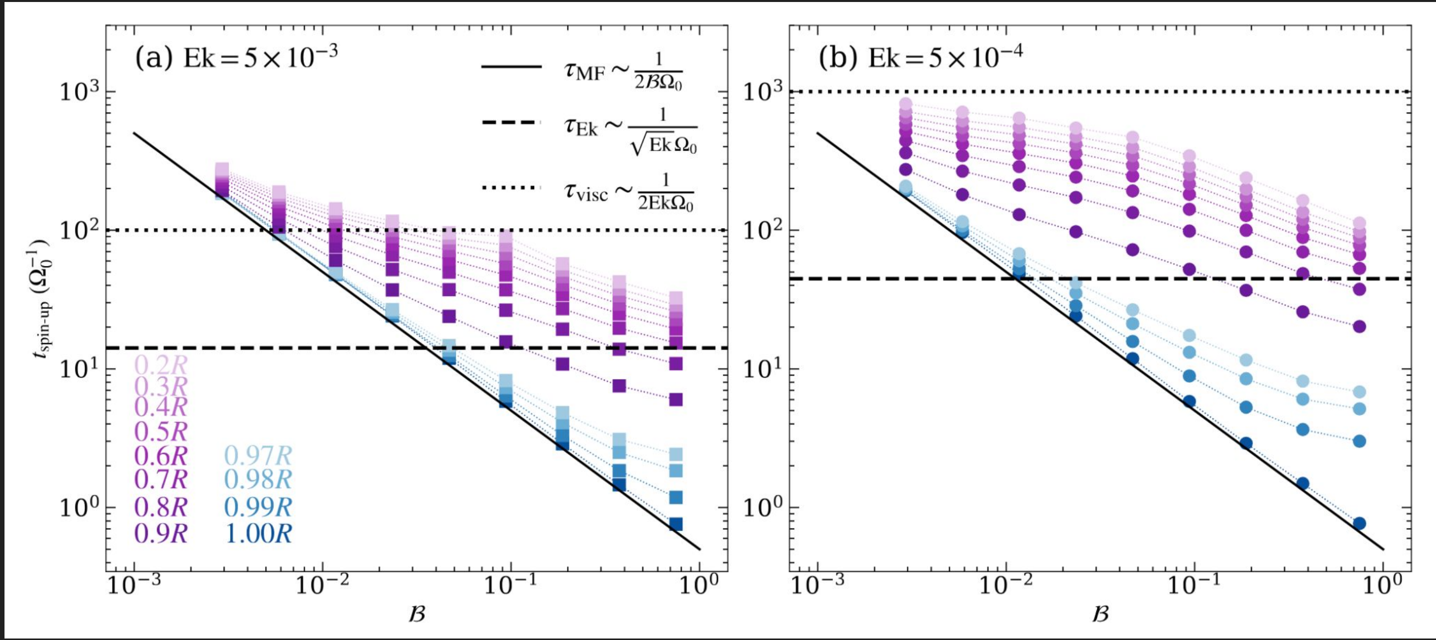
Spinning up a two-component fluid IV

- Spin-up timescales for two different Ekman numbers:



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Spinning up a two-component fluid V

- Spin-up timescales for two different Ekman numbers: