Stellar Death
Part II --
Neutron Stars

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Image credit: ESO/L.Calçada
1. Formation

Image credit: ESO/L.Calçada
1. Formation - Progenitors

- Progenitors of NSs are **massive stars** with 8 - 20 M\(\odot\). As they move from the main sequence, **hydrogen** fuses into **helium**, which burns into **carbon**.

- For stars with M < 8M\(\odot\), the fusion process halts at this stage and stars cool down to form white dwarfs. Above this limit, temperatures and pressures are high enough to **ignite carbon**.

- Massive stars undergo subsequent burning stages and become **layered like onions** fusing increasingly higher mass elements. Chain halts at iron (no more net energy gain by fusion), leaving behind an **iron-nickel core**.
1. Formation - Core Collapse SN I

- The iron-nickel core experiences extreme gravitational pressure and is only supported by the electron degeneracy pressure.

- As outer layers continue to burn, the mass of the iron-nickel core grows. Once the Chandrasekhar mass of 1.4 $M_\odot$ is reached, electron degeneracy pressure is no longer sufficient and gravitational collapses is initiated.

- Outer core layers fall inwards at ~25% the speed of light. This increases the temperature and generates gamma rays that split the iron nuclei into helium and free neutrons (photodisintegration).

- As the density in the inner core increases, electron captures take place: $p + e^- \rightarrow n + \nu_e$. Neutrinos take away energy, accelerating the collapse.
1. Formation - Core Collapse SN II

- At nuclear densities, collapse is eventually **halted** by the neutron-neutron repulsion and their degeneracy pressure. Infalling matter **bounces off** and forms an outward propagating **shock front**.

- This shock wave interacts with the heavy nuclei, causing it to lose energy and **stall**. The stalled shock is **revived** because of the **neutrinos** generated in the inner core (1% required). It is not clear how this exactly works (!).

- The revived shock front hits the outer layers and blast away material, which we observe as a **supernova**. Left behind is a **tiny compact object**.
1. Formation - Simulations

- In reality, the problem is much more complex and requires:
  - 3D magnetohydrodynamics
    - rotation, magnetic fields, convection, instabilities
  - General relativity (GW emission)
  - Neutrino transport
  - Realistic microphysics
    - dense-matter equation of state, neutrino interactions
  - Main problem: reviving the shock

- Such simulations are very (!) computationally expensive and complicated.
1. Formation - Observations

- Supernovae are the most energetic events in the Universe, releasing about $10^{51}$ erg. For comparison, a hydrogen bomb releases about $10^{24}$ erg.

- Stars become about 8 orders of magnitude brighter and are visible for months afterwards.

- We also observe supernova remnants, large-scale structures created by material ejected during the explosion and the interstellar medium accumulated by the shock.

Cas A
SN ~1670,

Image credit: NASA, JPL-Caltech, STScI, CXC, SAO

1.5 pc ~ $10^{13}$ km

SN1987a, Image credit: ESO
2. Interiors

Image credit: ESO/L.Calçada
2. Interiors - Structure

- During the collapse, matter is compressed so much that repulsive forces between the protons and electrons are overcome and **neutrons** created. NSs contain about 95% neutrons, hence the name.

- Compact remnants have **radii** of the order of ~10 km and **masses** of ~1.4 - 2 M⊙. This gives **mass densities** of up to 10^{15} g/cm^3, exceeding those of atomic nuclei.

- We do not (!) know how matter behaves at these ultra-high densities. The **equation of state** of dense nuclear matter is **unknown**, but there is a **canonical understanding** of the NS structure.

Watts et al. (2016)
2. Interiors - TOV Equation

- A simple model of NSs can be obtained by assuming that the star is a spherically symmetric body of isotropic material, which is in **static gravitational equilibrium** and correctly described by General Relativity.

- Such an object can be described by the **metric** (line element)

\[ ds^2 = e^\nu c^2 dt^2 - \left( 1 - \frac{2Gm}{rc^2} \right)^{-1} dr^2 - r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \]

where \( \nu = \nu(r) \) and \( m = m(r) \) is the **gravitational mass** within a radius \( r \).

- Solving Einstein’s equation for this metric leads to the **Tolman-Oppenheimer-Volkoff (TOV)** equation

\[ \frac{dP}{dr} = -\frac{Gm}{r^2} \rho \left( 1 + \frac{P}{\rho c^2} \right) \left( 1 + \frac{4\pi r^3 P}{mc^2} \right) \left( 1 - \frac{2Gm}{rc^2} \right)^{-1} \]

with density \( \rho \) and pressure \( P \). The **continuity equation** provides

\[ \frac{dm}{dr} = 4\pi r^2 \rho \]

\[ M = m(R) = \int_0^R 4\pi r^2 \rho \, dr \]
2. Interiors - Maximum Mass

- In the **Newtonian limit**, we would recover the standard equations that describe a spherical object in hydrostatic equilibrium.

- In general, to solve the TOV equation, we require an **equation of state**, a relation that connects pressure and density, \( P = P(\rho) \), but an analytical solution is possible for an incompressible star with constant density \( \rho = \rho_c \).

- In this case, the **mass function** is \( m(r) = \frac{4\pi r^3 \rho}{3} \) for \( r \leq R \). Integration gives

\[
P(r) = \rho_c c^2 \frac{\sqrt{1 - 2GM/c^2 R} - \sqrt{1 - 2GMr^2/c^2 R^3}}{\sqrt{1 - 2GMr^2/c^2 R^3} - 3\sqrt{1 - 2GM/c^2 R}}
\]

- Physical solutions require a **finite pressure** \( P \) at the centre \( r = 0 \)

\[
1 = 3\sqrt{1 - 2GM/c^2 R}, \quad \rightarrow \quad R > \frac{9GM}{4c^2}
\]

which results in a maximum mass (for \( \rho_c = 5 \times 10^{14} \text{ g/cm}^3 \))

\[
M < M_{\text{max}} \equiv \sqrt{\frac{1}{3\pi G \rho_c}} \frac{4c^3}{9G} \approx 5M_\odot
\]
2. Interiors - M/R Relations

- The maximum mass above which NSs become unstable is **not unique**, but depends on the equation of state. This is often illustrated as

- Equations of state are typically classified as
  - **Soft**: matter is more compressible and allows for larger central densities and subsequently smaller radii and maximum masses
  - **Stiff**: smaller central densities with larger radii and maximum masses

- **Observations** of neutron star masses can put **stringent limits** on theoretical equations of state calculations.

Demorest et al. (2010)
3. Pulsars

Image credit: ESO/L.Calçada
3. Pulsars - Rotation and Fields

- NSs also have incredibly high magnetic fields in the range of $10^8 - 10^{15}$ G. For comparison, the Earth’s field is about 0.5 G.

- NSs are also very fast and stable rotators (up to ~700 times per second).

- As rotation and magnetic field axes are misaligned, NSs emit pulses similar to a lighthouse.

Image credit: J. Christiansen
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- Pulses are observable with radio telescopes on Earth. First detection of a pulsar by Jocelyn Bell Burnell in 1967, over 30 years after their existence was first predicted.
3. Pulsars - Dispersion

- As the electromagnetic radiation propagates from the pulsar towards the Earth, the waves interact with electrons in the interstellar medium. This causes **dispersion**: lower frequencies are delayed w.r.t. higher frequencies.

Video credit: CAASTRO
3. Pulsars - Dispersion Measure

- Dispersion causes a characteristic **frequency dependence** and the **delay** between two frequencies is given by

\[ \Delta t = k_{DM} \cdot DM \cdot \left( \frac{1}{\nu_{lo}^2} - \frac{1}{\nu_{hi}^2} \right) \]

\[ k_{DM} = \frac{e^2}{2\pi m_e c} \approx 4.149 \text{ GHz}^2 \text{ pc}^{-1} \text{ cm}^3 \text{ ms} \]

- **DM** is the **dispersion measure** (integrated e\(^-\) density), measured in pc per cm\(^3\)

Image credit: E. Parent

**observed dispersed pulse**

**de-dispersed pulse (DM=20pc/cm\(^3\))**
3. Pulsars - Galactic distribution

- To date, we know of ~3,000 pulsars.

- We can combine their measured DM values with theoretical models of the electron distribution in our galaxy to estimate distances to the pulsars.

- Important tool to test the distribution of the interstellar medium.

Image credit: E. Parent
3. Pulsars - P-Pdot Diagram

- Because pulsars are incredibly stable rotators, we can time them and not only measure **periods** but also **period derivatives** (for ~2,500 objects).

- Plot these in a P-Pdot diagram to study different **classes** of the pulsar population and their characteristics, e.g. ages and magnetic fields.

- We can estimate

\[ \tau_c \sim 0.5 \frac{P}{\dot{P}}^{-1} \]

\[ B \sim 3.2 \times 10^{19} \left( \frac{P}{\dot{P}} \right)^{1/2} \text{ G} \]

Data from the ATNF pulsar catalogue
3. Pulsars - Glitches

- Because pulsar timing is very precise, we can detect very small changes in the stars’ rotation. For ~200 pulsars, we have detected glitches, sudden spin ups that interrupt the regular neutron star spin down.

- These spin ups are typically attributed to internal physics and can be explained by a superfluid component. This superfluid is decoupled from the crust (which we observe via timing) until a critical lag is reached. Angular momentum is transferred and the crust spun up.

- By analysing the morphology of glitches, we can study dense matter.
4. Gravitational Waves

Image credit: ESO/L.Calçada
4. GWs - Interferometers

- Gravitational waves are **ripples in space-time**, generated by acceleration of masses close to the speed of light. They were predicted by Einstein in 1916 based on his theory of General Relativity.

- Even for the most extreme objects (NSs and BHs) these ripples are of very **small amplitude** and detection require highly **sensitive interferometers**.

- The **first detection** of GWs from a binary BH merger was made in 2015, almost 100 years after their first prediction.
4. GWs - Isolated NSs

- Due to their compactness, NSs have strong gravity ($\sim 10^{11} \times$ the Earth’s gravitational acceleration). These compact objects interact strongly with space-time and are major sources of detectable gravitational waves.

- GWs are emitted by systems that have a non-vanishing mass quadrupole moment. Isolated NSs can thus emit gravitational waves because of ‘mountains’ on their surfaces or internal oscillations of the fluids.

- Resulting GWs are of small amplitude resulting in GW strains $h_0 \lesssim 10^{-24}$. This is below the sensitivity of current detectors - continuous gravitational waves from isolated NSs have not been detected (yet!).

- **Non-detection** of GWs from pulsars set limits on their ellipticities

\[ \epsilon = 9.5 \times 10^{-5} \left( \frac{h_0}{1.2 \times 10^{-24}} \right) \left( \frac{D}{1 \text{ kpc}} \right) \left( \frac{100 \text{ Hz}}{f} \right)^2 \]

LSC (2019)

- For some NSs, this constraint limits the size of mountains on their surfaces to less than 1mm.
4. GWs - Binary NSs

- A **binary neutron star system** produces much stronger space-time disturbances and GW signals, because the large orbital energy of the binary is converted into gravitational waves.

- As the system emits gravitational radiation, the **orbital period** decreases. If one of the NSs is a pulsar, we can accurately measure the orbital decay.

- This was first achieved in the **Hulse-Taylor pulsar** (PSR B1913+16): first **indirect detection** of gravitational waves that showed that **GR** is correct to high precision in strong regime.

![Diagram of neutron star system](image)

- ~7.75 h
- ~1.39 M
- ~60 ms
- ~1.44 M
- ~3x10^6 km

- **Weisberg and Taylor (2004)**

- Merge in 3x10^8 yr
4. GWs - GW170817

- Because NSs are so complex and we do not understand all the microphysics (they have ‘hair’), it is very difficult to accurately predict the wave forms.

- First direct GW detection from two merging NSs was made two years ago. Signal was very bright not only in GW but also had EM counterparts in $\gamma$-rays (sGRB detected by Fermi ~2 s after GW signal), X-rays, optical, radio and IR.

- This event initiated the era of multi-messenger astronomy: learn about astrophysics from many different angles.
The End!

Image credit: ESO/L.Calçada
Reading

- *Black Holes, White Dwarfs, and Neutron Stars* - Shapiro and Teukolsky
- *Neutron Stars 1: Equation of State and Structure* - Haensel, Potekhin and Yakovlev
- *Handbook of Pulsar Astronomy* - Lorimer and Kramer
- *Neutron Stars and Pulsars* - Becker (Ed.)
- *Gravitation* - Misner, Thorne and Wheeler
- *General Relativity* - Wald

Image credit: ESO/L.Calçada