Superfluids

and Neutron Stars

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PHYS 432

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What is a Superfluid?

From *Absolute Zero*, BBC Four Documentary (2008)

https://www.youtube.com/watch?v=2Z6UJbwxBZI

Discovery

- **Liquid helium** was discovered by Onnes in 1908. While not observing a superfluid transition he used the helium to cool metals to very low temperatures and found that their electrical resistance disappeared $\rightarrow$ discover superconductivity in 1911.
- Below the 4.21 K boiling point, helium-4 behaves like ordinary liquids with small viscosities. But it does **not solidify** at lower temperatures and normal pressures. Instead, at **2.171 K** helium undergoes a transition into a **new fluid phase**.
- New phase first detected by Kapitsa, Allen and Misener in 1937 as a characteristic change in the specific heat capacity. The observed behaviour resembled the Greek letter λ $\rightarrow$ transition at the **Lambda point**.

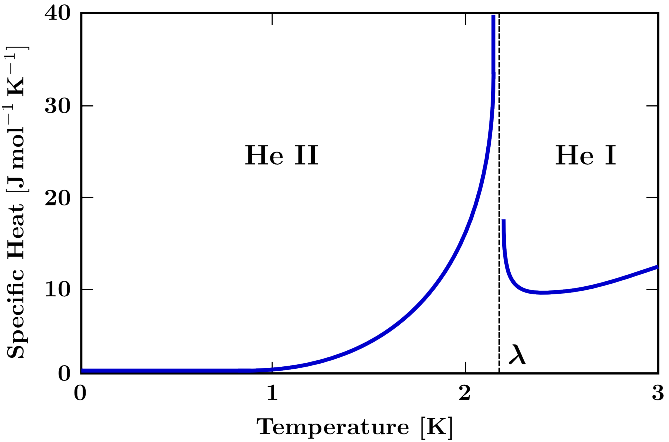


Figure 1: The superfluid transition in helium-4 at the 2.171 K Lambda point.

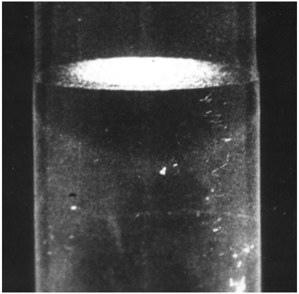
Contradicting Behaviour

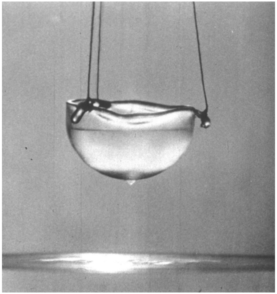
- Kaptisa observed flow through a μm sized tube below the Lambda point (implying a very low viscosity) and coined the term **superfluidity** in analogy with superconductivity (although the connection was not understood).
- Other experimental results: dragging an object through helium II showed non-viscous behaviour, while oscillations of a torsion pendulum were damped and revealed viscous characteristics $\rightarrow$ **contradicting behaviour??**
- Tisza solved this problem in 1938 by introducing a **phenomenological two-fluid model** $\rightarrow$ helium II is a mixture of two physically inseparable fluids: one flows without friction and one has ordinary viscosity.

Two-fluid Phenomenology

- Two-fluid model explains the features observed in the video, such as creeping up the container walls and the fountain effect.

Figure 2: Various effects in superfluid helium II.





Landau's Two-fluid Model

- Landau improved the two-fluid model in the 1940s by providing a **semi-microphysical explanation** $\rightarrow$ At $T = 0$, a fluid is in a perfect, frictionless state, i.e. **superfluid**.
- For $T > 0$, excitation of phonons and other quasi-particles (rotons) takes place. These thermal **excitations** behave like an ordinary gas, are responsible for the transport of heat and form the viscous fluid component.
- Based on his ideas, Landau suggested an **experiment** to measure the fraction of superfluid in helium $\rightarrow$ performed by Andronikashvili in 1946.

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- Based on his ideas, Landau suggested an **experiment** to measure the fraction of superfluid in helium $\rightarrow$ performed by Andronikashvili in 1946.

Question: Can you think of an experiment?

Superfluid Fraction

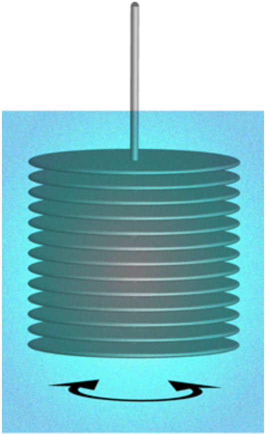

Figure 3: Set-up of Andronikashvili's experiment.

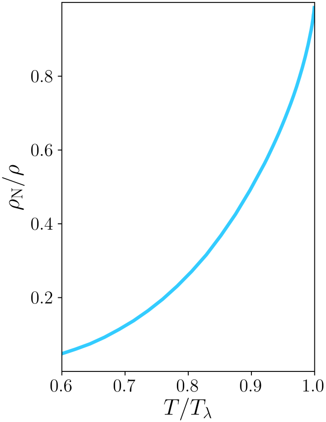

Figure 4: Normal fluid fraction in helium II.

Quantum Mechanics

- At $T = 0$, helium II is completely superfluid $\rightarrow$ **ground state**.
- F. London was the first to suggest that bosonic helium-4 atoms could turn superfluid by **Bose-Einstein condensation** $\rightarrow$ identical particles with integer spin follow Bose-Einstein statistics and are allowed to share the same quantum state.
- As $T \rightarrow 0$ they tend to occupy the lowest accessible quantum state, resulting in a new phase: the Bose-Einstein condensate (BEC). This is a **macroscopic quantum phenomenon**.
- In the case of helium II, the Lambda point reflects the onset of this condensation into a new state.

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Particle Statistics

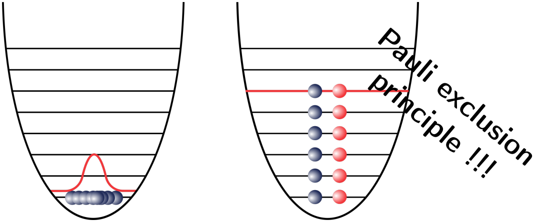

Figure 5: Quantum mechanical ground state of bosons and fermions in a harmonic-oscillator potential.

- Number of particles n_i with energy ε_i for B-E/F-D statistics

$$n_i(\varepsilon_i) = \frac{g_i}{e^{(\varepsilon_i-\mu)/k_\mathrm{B}T} - 1}, \qquad n_i(\varepsilon_i) = \frac{g_i}{e^{(\varepsilon_i-\mu)/k_\mathrm{B}T} + 1}, \qquad (1)$$

where g_i is the degeneracy level, μ the chemical potential, k_B the Boltzmann constant and T the temperature.

Wave Function

- Use QM to model the superfluid component. Ground state is characterised by a single **macroscopic wave function**, which represents the superposition of all individual superfluid states:

$$\Psi(\mathbf{r}, t) = \Psi_0(\mathbf{r}, t)\, e^{i\varphi(\mathbf{r}, t)}, \tag{2}$$

where $\Psi_0(\mathbf{r}, t)$ and $\varphi(\mathbf{r}, t)$ are the real amplitude and phase.

- $\Psi(\mathbf{r}, t)$ is the solution to the **Schrödinger equation**,

$$i\hbar \frac{\partial \Psi(\mathbf{r}, t)}{\partial t} + \frac{\hbar^2}{2m_\mathrm{c}} \nabla^2 \Psi(\mathbf{r}, t) - \mu(\mathbf{r}) \Psi(\mathbf{r}, t) = 0, \tag{3}$$

with the reduced Planck constant $\hbar$, the fluid's chemical potential $\mu(\mathbf{r})$ and the mass m_c of one bosonic particle.

QM vs Hydro

- The absolute value of the wave function is $|\Psi|^2 \equiv \Psi\Psi^*$ (where $*$ denotes the complex conjugate) $\rightarrow$ amplitude is related to the **number density** n_c of bosons occupying the ground state

$$|\Psi(\mathbf{r}, t)|^2 = |\Psi_0(\mathbf{r}, t)|^2 = n_\mathrm{c}(\mathbf{r}, t). \tag{4}$$

We can connect the abstract quantum mechanical description to a **hydrodynamical formalism**, i.e. the averaged behaviour of a large number of particles on macroscopic scales.

- Use a **Madelung transformation** to do this $\rightarrow$ substitute $\Psi(\mathbf{r}, t)$ into SE and separate the real and imaginary part.

Madelung Transformation

- Obtain two coupled equations of motion for Ψ_0 and φ:

$$\hbar \frac{\partial \varphi}{\partial t} + \frac{\hbar^2}{2m_c} (\nabla \varphi)^2 + \mu - \frac{\hbar^2}{2m_c \Psi_0} \nabla^2 \Psi_0 = 0, \qquad (5)$$

$$\frac{\partial \Psi_0}{\partial t} + \frac{\hbar}{2m_c} \left(2\nabla \Psi_0 \cdot \nabla \varphi + \Psi_0 \nabla^2 \varphi\right) = 0. \qquad (6)$$

- Multiplying (6) with Ψ_0 and using the chain rule gives

$$\frac{\partial |\Psi_0|^2}{\partial t} + \frac{\hbar}{m_c} \nabla \cdot \left(|\Psi_0|^2 \nabla \varphi\right) = 0. \qquad (7)$$

Question: Does this look familiar?

Continuity Equation

- It is equivalent to the **continuity equation** of fluid mechanics

$$\boxed{\frac{\partial \rho_{\mathsf{s}}}{\partial t} + \nabla \cdot \mathbf{j}_{\mathsf{s}} = 0,} \tag{8}$$

when using the superfluid mass density, $\rho_{\mathsf{s}} = m_{\mathrm{c}} n_{\mathrm{c}}$, and the quantum mechanical momentum density,

$$\mathbf{j}_{\mathsf{s}} = \frac{i\hbar}{2}\left[\Psi \nabla \Psi^* - \Psi^* \nabla \Psi\right] = \hbar |\Psi_0|^2 \nabla \varphi, \tag{9}$$

- Identify $\mathbf{j}_{\mathsf{s}} \equiv \rho_{\mathsf{s}} \mathbf{v}_{\mathsf{s}}$ to define a **superfluid velocity**

$$\mathbf{v}_{\mathsf{s}} \equiv \frac{\hbar}{m_{\mathrm{c}}} \nabla \varphi. \tag{10}$$

Euler Equation

- Take the gradient of (5) and use irrotationality to find

$$\frac{\partial \mathbf{v}_\mathrm{s}}{\partial t} + \left(\mathbf{v}_\mathrm{s} \cdot \nabla\right) \mathbf{v}_\mathrm{s} = -\nabla \tilde{\mu} + \nabla \left(\frac{\hbar^2}{2 m_\mathrm{c}^2 \sqrt{n_\mathrm{c}}} \nabla^2 \sqrt{n_\mathrm{c}} \right), \quad (11)$$

  where $\tilde{\mu} \equiv \mu / m_\mathrm{c}$ is the specific chemical potential $\rightarrow$ this resembles the **Euler equation** of an ideal fluid apart from the second term on the right hand side.

- This **quantum pressure** term reflects the quantum nature of the system. It is negligible if the spatial variations of Ψ_0 occur on large scales. This give the **standard momentum equation**:

$$\boxed{\frac{\partial \mathbf{v}_\mathrm{s}}{\partial t} + \left(\mathbf{v}_\mathrm{s} \cdot \nabla\right) \mathbf{v}_\mathrm{s} + \nabla \tilde{\mu} = 0.} \quad (12)$$

Two-fluid Equations I

- For $T > 0$, two components denoted by indices 'S' and 'N':

$$\rho = \rho_\mathrm{N} + \rho_\mathrm{S}, \qquad \mathbf{j} = \rho_\mathrm{N}\mathbf{v}_\mathrm{N} + \rho_\mathrm{S}\mathbf{v}_\mathrm{S}. \tag{13}$$

- If coupling and dissipation can be neglected, we can derive **two continuity equations**. One for the total mass density

$$\boxed{\frac{\partial\rho}{\partial t} + \nabla\cdot\mathbf{j} = 0} \tag{14}$$

and one for the entropy per unit mass s (which is transported with the normal fluid component)

$$\boxed{\frac{\partial(\rho s)}{\partial t} + \nabla\cdot(\rho s\mathbf{v}_\mathrm{N}) = 0.} \tag{15}$$

Two-fluid Equations II

- For incompressible flow, $\nabla \cdot \mathbf{v}_{\mathsf{S}} = \nabla \cdot \mathbf{v}_{\mathsf{N}} = 0$, **momentum conservation equations** for each component can be derived:

$$\rho_{\mathsf{S}} \left[\frac{\partial \mathbf{v}_{\mathsf{S}}}{\partial t} + \left(\mathbf{v}_{\mathsf{S}} \cdot \nabla \right) \mathbf{v}_{\mathsf{S}} \right] + \frac{\rho_{\mathsf{S}}}{\rho} \nabla p - \rho_{\mathsf{S}} s \nabla T = 0, \quad (16)$$

$$\rho_{\mathsf{N}} \left[\frac{\partial \mathbf{v}_{\mathsf{N}}}{\partial t} + \left(\mathbf{v}_{\mathsf{N}} \cdot \nabla \right) \mathbf{v}_{\mathsf{N}} \right] + \frac{\rho_{\mathsf{N}}}{\rho} \nabla p + \rho_{\mathsf{S}} s \nabla T - \eta \nabla^2 \mathbf{v}_{\mathsf{N}} = 0. \quad (17)$$

- First equation describes the inviscid ground state superfluid (responsible for frictionless dynamics). Second equation is the equation of motion for the normal constituent (composed of elementary excitations), which resembles a classical Navier-Stokes fluid of viscosity η.

Classical Rotation

- Consider a normal fluid confined inside a rotating vessel. It is said to follow **rigid-body rotation** when

$$\mathbf{v} = \boldsymbol{\Omega} \times \mathbf{r} \tag{18}$$

in the intertial frame. Here $\boldsymbol{\Omega}$ is the container's angular velocity vector and $\mathbf{r}$ the position vector.

- As a result of shearing, **vorticity** is created when the fluid is flowing past container walls. This is defined by

$$\boldsymbol{\omega} \equiv \nabla \times \mathbf{v} = 2\boldsymbol{\Omega}. \tag{19}$$

- Vorticity transport is described by a diffusion equation.

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Question: Is it possible to rotate a superfluid?

- We saw earlier that $\mathbf{v}_s \propto \nabla\varphi$. Taking the curl gives

$$\boldsymbol{\omega}_s = \nabla \times \mathbf{v}_s = 0. \tag{20}$$

- **Superflow is irrotational**. This implies that for a smooth velocity field, $\mathbf{v}_s$, the circulation around an arbitrary contour $\mathcal{L}$ vanishes because of Stoke's theorem:

$$\Gamma = \oint_{\mathcal{L}} \mathbf{v}_s \cdot d\mathbf{l} = \int_{\mathcal{A}} (\nabla \times \mathbf{v}_s) \cdot d\mathbf{S} = 0. \tag{21}$$

- The superfluid component cannot develop circulation in a classical manner. However, experiments in the 1960s showed solid body rotation like classical fluid. So the question is **how**?

Quantised Vortices I

- Solve this problem by introducing **vortices** to quantise the circulation:

$$\Gamma = \oint_{\mathcal{L}} \mathbf{v}_{\mathrm{s}} \cdot d\mathbf{l} = \frac{\hbar}{m_c} \oint_{\mathcal{L}} \nabla\varphi \cdot d\mathbf{l}$$

$$= \frac{h}{m_c} n \equiv \kappa n, \qquad n \in \mathbb{Z}. \qquad (22)$$

- κ is the **quantum of circulation**.
- Solving (22) in cylindrical coordinates $\{r, \theta, z\}$ gives for the superfluid velocity

$$\mathbf{v}_{\mathrm{s}}(r) = \frac{\Gamma}{2\pi r}\hat{\boldsymbol{\theta}}. \qquad (23)$$

Figure 6: Envisage vortices as tiny, rapidly rotating tornadoes.

Quantised Vortices II

- The idea of quantisation was pioneered by Onsager and Feynman and improved by Abrikosov in 1957. He calculated that vortices arrange themselves in a **hexagonal array**.

- The circulation of all vortices mimics rotation on macroscopic lengthscales. Therefore, the **vortex area density**, $\mathcal{N}_{\mathrm{v}}$, is proportional to the total number of vortices per unit area,

$$\boldsymbol{\omega} = 2\boldsymbol{\Omega} = \mathcal{N}_{\mathrm{v}}\kappa\hat{\boldsymbol{z}}. \tag{24}$$

- Helium II rotating at $\Omega = 1\,\mathrm{rad\,s^{-1}}$ has $\mathcal{N}_{\mathrm{v}} \approx 10^4\,\mathrm{cm^{-2}}$.

- For a regular array, we also obtain the **intervortex distance**:

$$d_{\mathrm{v}} \simeq \mathcal{N}_{\mathrm{v}}^{-1/2} = \left(\frac{\hbar\pi}{\Omega m_{\mathrm{c}}}\right)^{1/2} \approx 0.1\,\mathrm{mm} \qquad \text{for helium II.} \tag{25}$$

Solid-body Rotation

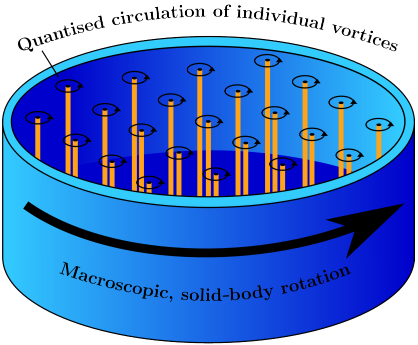

Figure 7: Different to a viscous fluid, a superfluid minimises its energy by forming a regular vortex array, aligned with the rotation axis.

Question: How can we spin-up or spin-down a superfluid?

Solid-body Rotation

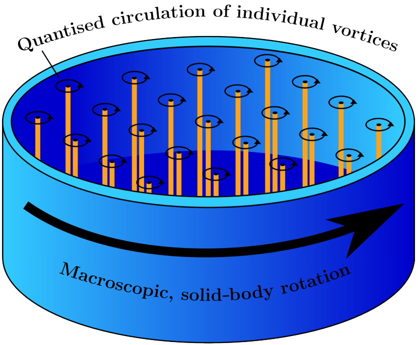

Figure 7: Different to a viscous fluid, a superfluid minimises its energy by forming a regular vortex array, aligned with the rotation axis.

A change in angular momentum is accompanied by the creation (spin-up) or destruction (spin-down) of vortices.

Mutual Friction

- Vortices interact with the viscous fluid component which causes dissipation. This coupling mechanism is called **mutual friction** $\rightarrow$ responsible for spinning up the superfluid fraction.

- In the 1960s, Hall and Vinen realised that dissipation arises due to the collision of excitations and vortex cores. For a helium II sample rotating at $\mathbf{\Omega} = \Omega\,\hat{\mathbf{\Omega}}$, they postulated

$$\mathbf{F}_{\mathrm{mf}} = \mathcal{B}\frac{\rho_{\mathsf{S}}\rho_{\mathsf{N}}}{\rho}\,\hat{\mathbf{\Omega}} \times (\mathbf{\Omega} \times \mathbf{w}_{\mathsf{SN}}) + \mathcal{B}'\frac{\rho_{\mathsf{S}}\rho_{\mathsf{N}}}{\rho}\,\mathbf{\Omega} \times \mathbf{w}_{\mathsf{SN}}, \tag{26}$$

  where $\mathbf{w}_{\mathsf{SN}} \equiv \mathbf{v}_{\mathsf{S}} - \mathbf{v}_{\mathsf{N}}$ is the relative velocity.

- The **dimensionless parameters** $\mathcal{B}$ and $\mathcal{B}'$ reflect the strength of the mutual friction and can be determined experimentally.

Mutual Friction Coefficients

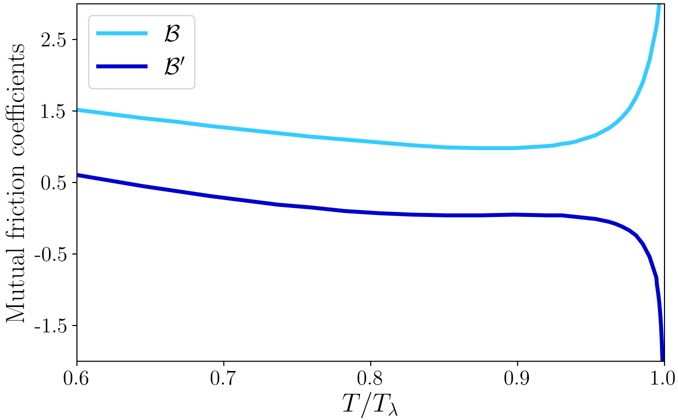

Figure 8: Dimensionless mutual friction coefficients in helium II as a function of temperature.

Vortex Tension

- Another force influences the superfluid dynamics: Vortices have a large self-energy and resist bending (comparable to the tension of a guitar string) $\rightarrow$ **tension** keeps vortices straight:

$$\mathbf{T} = \rho_\mathsf{s} \frac{\kappa}{4\pi} \ln\left(\frac{d_\mathrm{v}}{a}\right)(\boldsymbol{\omega} \cdot \nabla)\,\hat{\boldsymbol{\omega}}, \tag{27}$$

where a is the vortex core radius and $\boldsymbol{\omega} \equiv \omega\,\hat{\boldsymbol{\omega}}$ with $\hat{\boldsymbol{\omega}} = \hat{\boldsymbol{\Omega}}$.

- Accounting for vortex curvature, the **mutual friction force** is

$$\boxed{\mathbf{F}_\mathrm{mf} = \mathcal{B}\frac{\rho_\mathsf{S}\rho_\mathsf{N}}{2\rho}\,\hat{\boldsymbol{\omega}} \times \left(\boldsymbol{\omega} \times \mathbf{w}_\mathsf{SN} - \frac{\mathbf{T}}{\rho_\mathsf{S}}\right) + \mathcal{B}'\frac{\rho_\mathsf{S}\rho_\mathsf{N}}{2\rho}\left(\boldsymbol{\omega} \times \mathbf{w}_\mathsf{SN} - \frac{\mathbf{T}}{\rho_\mathsf{S}}\right).} \tag{28}$$

- If the fluid velocities $\mathbf{v}_\mathsf{S}, \mathbf{v}_\mathsf{N}$ are no longer small, additional **dissipative terms** have to be included into the momentum equations. This results in the coupling of the two components.
- Accounting for mutual friction and vortex tension, one obtains the Hall-Vinen-Bekarevich-Khalatnikov **(HVBK) equations**:

$$\boxed{\rho_\mathsf{S}\left[\frac{\partial \mathbf{v}_\mathsf{S}}{\partial t} + (\mathbf{v}_\mathsf{S}\cdot\nabla)\,\mathbf{v}_\mathsf{S}\right] + \frac{\rho_\mathsf{S}}{\rho}\nabla p - \rho_\mathsf{S} s \nabla T - \frac{\rho_\mathsf{S}\rho_\mathsf{N}}{2\rho}\nabla \mathbf{w}_\mathsf{SN}^2 = \mathbf{T} + \mathbf{F}_{\mathrm{mf}},} \tag{29}$$

$$\boxed{\rho_\mathsf{N}\left[\frac{\partial \mathbf{v}_\mathsf{N}}{\partial t} + (\mathbf{v}_\mathsf{N}\cdot\nabla)\,\mathbf{v}_\mathsf{N}\right] + \frac{\rho_\mathsf{N}}{\rho}\nabla p + \rho_\mathsf{S} s \nabla T - \eta\nabla^2\mathbf{v}_\mathsf{N} + \frac{\rho_\mathsf{S}\rho_\mathsf{N}}{2\rho}\nabla \mathbf{w}_\mathsf{SN}^2 = -\mathbf{F}_{\mathrm{mf}}.} \tag{30}$$

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Any questions so far?

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Formation

- Neutron stars are one type of **compact remnant**. They are created during the final stages of stellar evolution.

- When a massive star of $8 - 30\,M_\odot$ ($M_\odot$ is the mass of the sun) runs out of fuel, it explodes in a **core-collapse supernova**.

- These explosions are some of the **most energetic events** in our Universe and can even be visible from the Earth.

- Exact physics are not understood and simulations are very expensive.

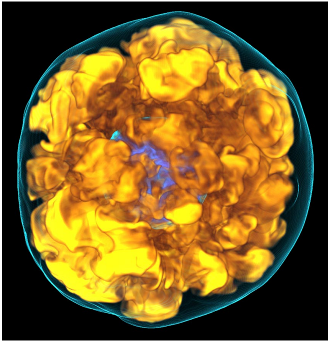

Figure 9: Snapshot of a modern 3D core-collapse supernova simulation.

General Properties

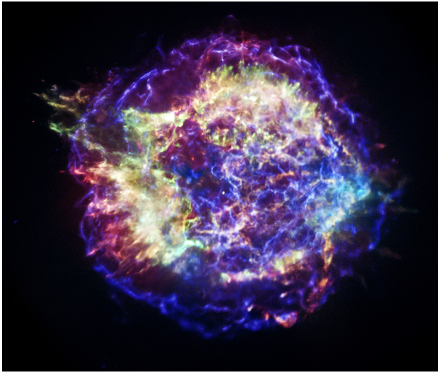

Figure 10: Chandra X-ray observation of the Cassiopeia A supernova remnant, which host the youngest known neutron star.

- During the collapse, matter is crushed so tightly together that gravity overcomes the repulsive force between electrons and protons. This **creates (a lot of) neutrons** via $p + e^- \rightarrow n + \nu_e$.
- Neutron stars typically have radii between $\mathbf{10-15\,km}$ and masses of $\mathbf{1.4-2\,M_\odot}$. This results in huge mass densities, $\boldsymbol{\rho \simeq 10^{15}\,gcm^{-3}}$ $\rightarrow$ exceeds the density of atomic nuclei.

Pulsars

- Additionally, neutron stars have incredibly high magnetic fields, i.e. $\mathbf{10^8 - 10^{15}\,G}$. For comparison, the Earth's magnetic field is about $0.5\,\mathrm{G}$.
- Because rotation and magnetic field axes are misaligned, neutron stars emit pulses similar to a **lighthouse**, which can be observed on Earth.
- **Pulsars** are very precise clocks and we measure their period.

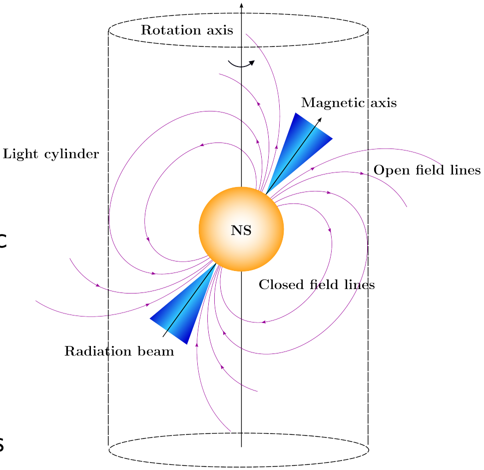

Figure 11: Sketch of the neutron star exterior.

$P\dot{P}$-diagram

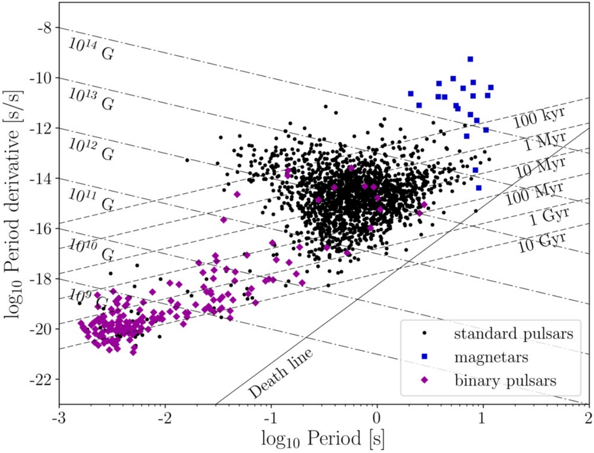

Figure 12: $P\dot{P}$ diagram of $\sim$ 2500 known radio pulsars. Different classes of neutron stars are shown.

Structure

- The interior neutron star structure is very complex and not well understood. However, there is a **canonical picture** of how they look like.
- After $\sim 10^4$ years neutron stars are in equilibrium and have temperatures of $10^6 - 10^8$ K. At this age, they are expected to contain **several layers**.
- Neutron stars have a **solid crust** and a **fluid core**.

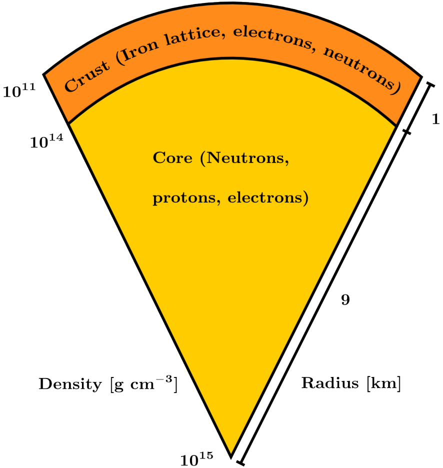

Figure 13: Sketch of the neutron star interior.

Superfluid Components?

- Neutron stars are very hot compared to low-temperature experiments (10^8 K vs. 2 K), but they are cold in terms of the nuclear physics. What does that mean?
- Neutrons are **fermions** and cannot undergo Bose-Einstein condensation. However, macroscopic quantum states can be created by the formation of **Cooper pairs**. This is described within the standard microscopic **Bardeen-Cooper-Schrieffer (BCS) theory** of superconductivity.
- Compare the equilibrium to the neutrons' **Fermi temperature**:

$$T_{\mathrm{F}} = k_{\mathrm{B}}^{-1} E_{\mathrm{F}} = 10^{12}\,\mathrm{K} \gg 10^{6} - 10^{8}\,\mathrm{K}. \tag{31}$$

Crustal and core neutrons form cosmic superfluids!!

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Objective

Any questions so far?

- As a result of the presence of **distinct components**, neutron stars can be modelled by means of a **multi-fluid formalism**. A set of equations similar to the HBVK equations for helium II capture their complex dynamics.
- It is **not possible to replicate** the extreme conditions present in neutron stars. However, one can try to use known laboratory analogues that are easy to manipulate in order to recreate and **study specific neutron star characteristics**. This way we can learn something about the physics of their interiors.

Helium II Spin-up

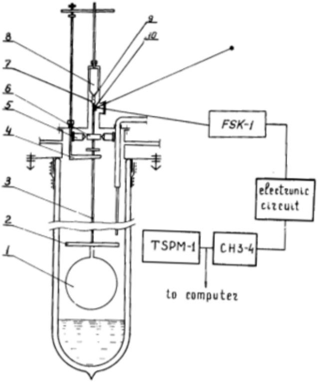

Figure 14: Schematic setup of the helium II spin-up experiments.

- First (and only) **systematic analysis** of rotating helium II by Tsakadze and Tsakadze in the 1970s, shortly after first observations of **glitches** in the Vela and Crab pulsar.
- Validate presence of superfluid components in neutron stars by measuring **relaxation timescales** after initial changes in the container's rotation.
- Performed for various temperatures, vessel configurations, initial angular velocities and velocity jumps.

'Lab NS' Helium II Glitch Analogy

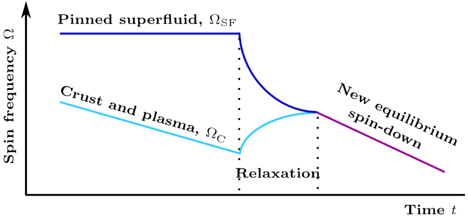

Figure 15: Sketch of an idealised neutron star glitch.

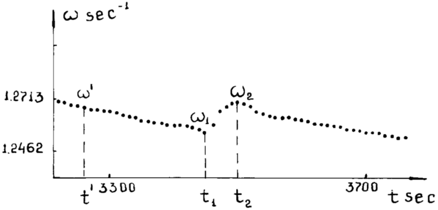

Figure 16: Measurement of a laboratory glitch.

- Glitches are **sudden spin-ups** that interrupt pulsar spin-down. Spontaneous acceleration also observed in rotating helium II.
- Dynamics well explained by a simple **two-component model**.
- However, the mechanism that causes coupling between crust and the pinned superfluid is not known → study with helium II.

Helium-3

- Superfluid behaviour below 3 mK. Transition is different to helium II: helium-3 atoms are fermions and have to form **Cooper-pairs**.
- Pairing in a spin-triplet, *p*-wave state: Cooper pairs have internal structure $\rightarrow$ **3 superfluid phases**.
- **B-phase** is similar to helium II.
- **A-phase** exhibits anisotropic behaviour and can form unusual vortex structures.

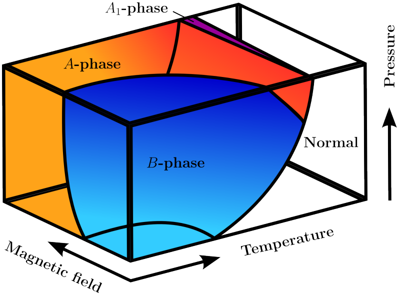

Figure 17: Phase diagram of helium-3.

Helium-3 Interfaces

- It is not understood how **interfaces** influence the neutron star dynamics $\rightarrow$ **crust-core transition** between two superfluids??

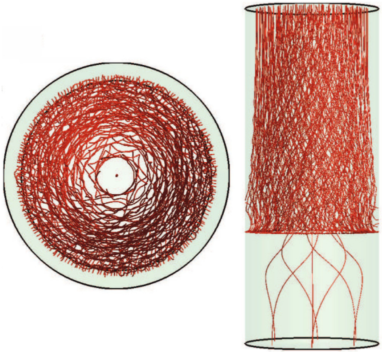

Figure 18: Vortex-line simulation for the spin-down of two-phase helium-3.

- Study vortices across an interface with rotating **two-phase samples** (different $\mathcal{B}$, $\mathcal{B}'$) using NMR measurements and modern vortex-line simulations.
- Interface strongly modifies dynamics.
  - **Vortex sheet** formation
  - **Vortex tangle** forms in B-phase, reconnections increase dissipation
  - **Differential rotation**

BECs

- A BEC of **weakly-interacting bosons** was first realised in 1995 by cooling Rubidum atoms to nanokelvin temperatures.

- **Superfluid transition** and vortex formation were observed in 1999.
- Very similar properties to helium II $\rightarrow$ governed by a generalised Schrödinger equation, the so-called **Gross-Pitaevskii equation**.
- **Absorption imaging** of clouds is a great advantage showing up to several hundred vortices $\rightarrow$ study behaviour of individual vortices.

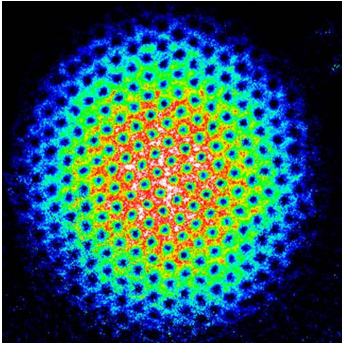

Figure 19: Vortex array in a rotating, dilute BEC of Rubidium atoms.

BEC Vortex Motion

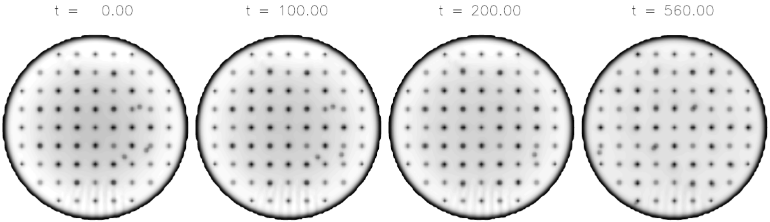

Figure 20: Snapshots of the superfluid density during the spin-down of a BEC in a cylinder.

- Time evolution of the Gross-Pitaevskii equation describes BEC **vortex motion** $\rightarrow$ use the same approach to study the pinned, decelerating **crustal superfluid** in neutron stars.
- Collective vortex motion in the presence of pinning potential can cause **glitch-like events** $\rightarrow$ study the unknown trigger.

BEC Quantum Turbulence

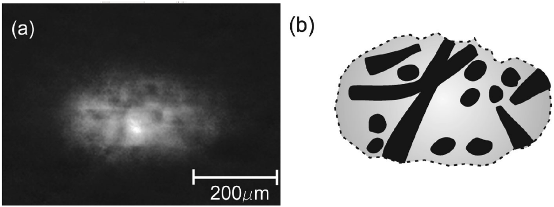

Figure 21: Vortex tangle in a BEC: snapshot of atomic density and the corresponding sketch.

- Chaotic superflow is referred to as **quantum turbulence**: the large scale features are similar to classical turbulence, but behaviour is different on small-scales $\rightarrow$ **take pictures** of this.
- Turbulence in neutron stars would alter the dissipation, which affects many macroscopic phenomena such as the post-glitch relaxation or oscillation damping $\rightarrow$ study **new phenomena**.

Conclusions

- Superfluids have the special ability to **flow without friction**, which leads to many surprising experimental results. This behaviour is a direct consequence of **quantum mechanics**. However, on large scales the hyrdodynamical features are well described within a simple **two-fluid model**.
- Neutron stars are born when massive stars run out of fuel and explode in **supernovae**. They contain a mass comparable to the Sun's within a radius of about ten kilometres and exhibit **extreme conditions**. Their interior is very difficult to probe.

There might be many exciting ways to combine both fields of research and probe the dynamics of the neutron star interior with superfluid laboratory experiments!!

Figure References

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- **Figure 2b:** https://commons.wikimedia.org/wiki/File:Liquid_helium_Rollin_film.jpg
- **Figure 2c:** http://pitp.physics.ubc.ca/archives/CWSS/showcase/topics/fluids.html
- **Figure 3:** https://physics.aps.org/articles/v3/51
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